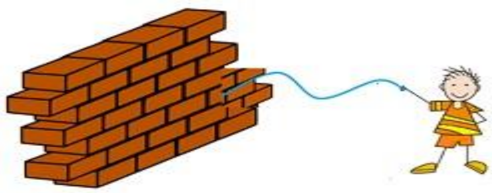
Head Branch: Dilshad colony delhi 110095.
First Branch: Shalimar garden UP 201006. And Second Branch : Jawahar park UP 201006
Contact No. 8750387081, 8700970941

Introduction

In this chapter we will see the importance of waves in our life.
We will also study about the different properties of waves, some terms related to waves and also about different types of waves.We will also learn how waves propagate.
For example: -

1. Medium required by the waves to travel from one point to another:-
    - Consider a boy holding a thread and one end of thread is tied to the wall.
    - When a boy moves the thread, the thread moves in the form of a wave.



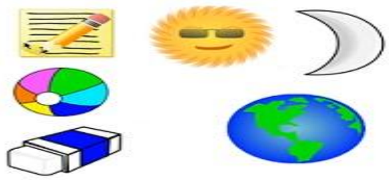
- Similarly a boat sailing over the sea,the boat is able to move because of waves.
- The ripples formed in a lake when we drop a stone in the lake.They are also waves.
- Earthquakes are caused due to the waves under the surface of the earth.
- The strings of the guitar when we play them are also waves again.
- Music system which we use to hear songs.This is due to sound waves.
- When 2 people talk they are able to hear each other because of the sound waves.

In the below Picture we can see waves need a medium to propagate.

2. Medium not required by some type of waves to move from one point to another:-
    - TV remote waves play important part.
    - Satelliteshelp ustouseTV, mobile phones, music system, the sun, the traffic lights, microwave,x-rays.

Some type of waves can propagate from one point to another without any medium.

3. Waves which are related to matter:-
   - There are some set of waves which are inside the matter.
   - For example: - whole of universe.



Waves propagating inside the matter

What is a wave?

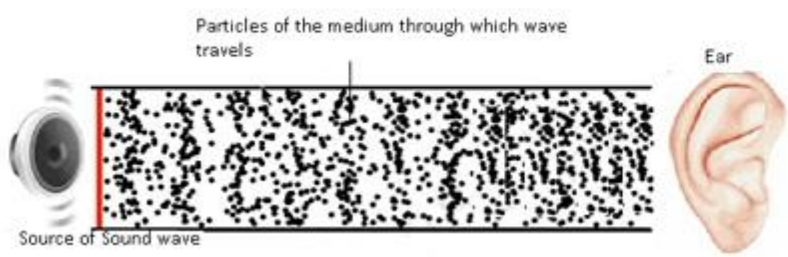
A wave is s disturbance that propagates through spaceand time,usually with transference of energy.

For example: -

- Consider the sound of the horn; this sound reaches our ear because of sound waves.
- There is transfer of energy from one point to another with the help of particles in the medium.
- These particles don't move they just move around their mean position,but the energy is getting transferred from one particle to another and it keeps on transferring till it reaches the destination.
- The movement of a particle is initiated by the disturbance.And this disturbance is transferred from one point to another through space and time.

Note:-Energy and not the matter is transferred from one point to another.

1. When a source of energy causes vibration to travel through the medium a wave is created.



Types of Waves

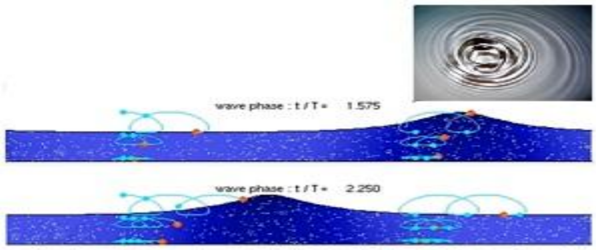
1. Mechanical waves
2. Electromagnetic waves
3. Matter waves

Mechanical waves: -

1. The mechanical waves are governed by all the Newton's laws of motion.
2. Medium is needed for propagation of the wave.

For Example: - Water Waves, Sound Waves

Water waves: They are mechanical waves for which a medium is required to propagate.



Sound waves: A guitar or music system. Sound waves need a medium to propagate.They cannot travel in vacuum.

Electromagnetic waves:-

- Electromagnetic waves are related to electric and magnetic fields.
- An electromagnetic wave, does not need a medium to propagate, it carries no mass,does carry energy.

Examples: - Satellite system, mobile phones,radio, music player, x-rays and microwave.

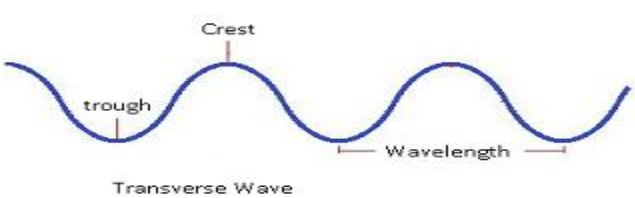
Matter waves:-

- Waves related to matter. Matter consists of small particles.
- Matter waves are associated with moving electrons,protons,neutrons & other fundamental particlesetc.
- It is an abstract concept.

Examples: - pencil, sun, moon, earth, ball, atoms.

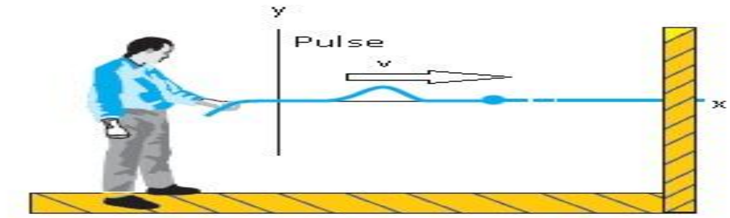
Transverse Waves

- The transverse waves are those in which direction of disturbance or displacement in the medium is perpendicular to that of the propagation of wave.
- The direction in which a wave propagates is perpendicular to the direction of disturbance.



For example:-

- Consider a manholding one end of a thread and other end of the threadis fixed to wall.
- When a little jerk is given to the thread in the upward direction.The entire thread moves in a wavy manner.
- The jerk propagated along the entire length of the thread.
- The small disturbance which came from the source at one end, that disturbance getting propagatedand that is known as direction of propagation.
- Disturbance is vertically upward and wave is horizontal.They are perpendicular to each other.
- This type of wave is known as transverse wave.

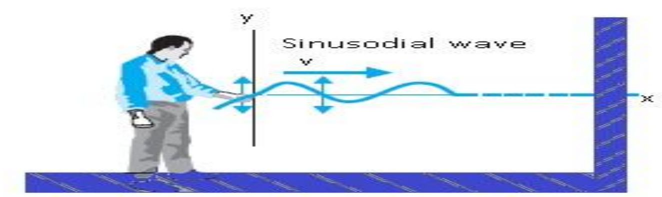


A single pulse is sent along a stretchedstring. A typical element of the string (suchas that marked with a dot) moves up andthen down as the pulse passes through.

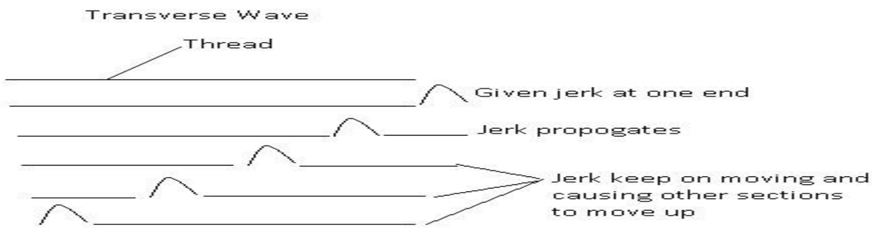
The element's motion is perpendicular to thedirection in which the wave travels.

<u>How are transverse waves caused?</u>

- When we pull a thread in upward direction the formation and propagation of the waves are possible because entire thread is under tension.
- This tension is the small disturbance which is given at one end and it gets transferred to its neighbouring molecules.
- This will keep on continuing.So this small pulse will get propagated along the length of the thread.
- The movement of the particles is perpendicular to the propagation of the wave and the wave will propagate horizontally.

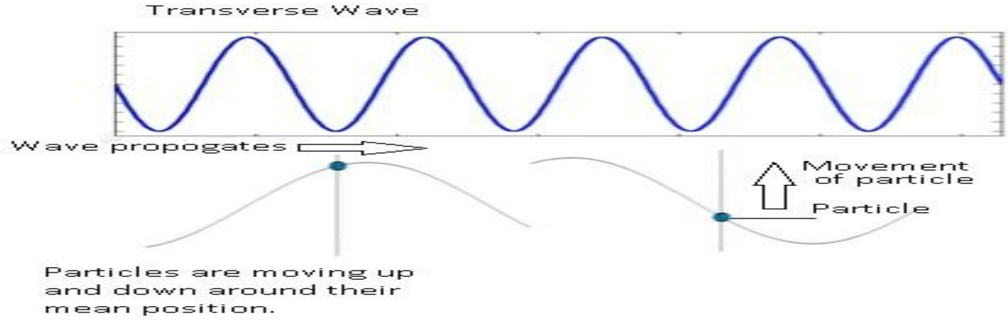


It is a transverse wave.
A sinusoidal wave is sent along the string.A typical element of the string moves up and down continuously as the wave passes.

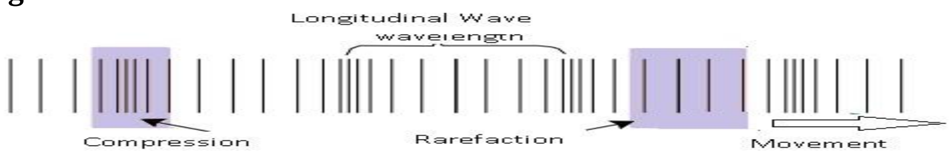


Conclusion: -

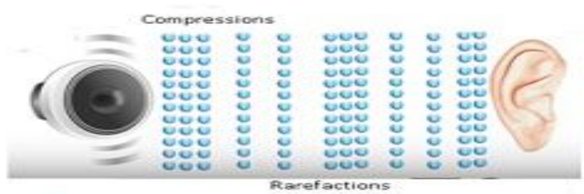
1. Transverse waves are those waves which propagates perpendicular to the direction of the disturbance.
2. Direction of disturbance is the direction of motion of particles of the medium.

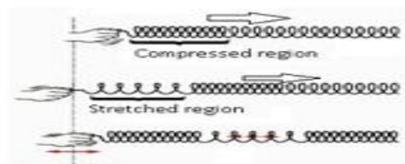


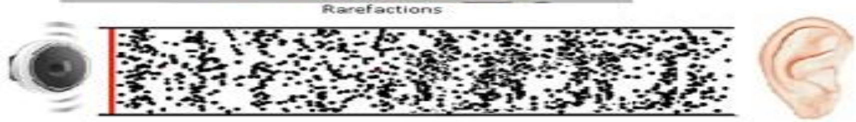
Longitudinal Waves



- Longitudinal means something related to length.
- In longitudinal waves direction of disturbance or displacement in the medium is along the propagation of the wave.







- For example: - Sound waves. Particles and wave moving along the horizontal direction. So both are in the same direction.
- In a Longitudinal wave there are regions where particles are very close to each other.These regions are known as compressions.
- In some regions the particles are far apart.Those regions are known as rarefactions.

Differentiate between transverse and longitudinal waves

Transverse	Longitudinal
Constituents of the medium oscillate perpendicular to the direction of the wave propagation.	Constituents of the medium oscillate parallel to the direction of wave propagation.
	

Displacement in a progressive wave

- Amplitude and phase together describe the complete displacement of the wave.
- Displacement function is a periodic in space and time.
- Displacement of the particles in a medium takes place along the y-axis.
- Generally displacement is denoted as a function of X and T, but here it is denoted by y.
- In case of transverse wave displacement is given as:
- y(x,t) where x=propagation of the wave along x-axis, and particles oscillates along y-axis.
- Therefore **$y(x,t)= A \sin(kx – \omega t + \varphi)$.**This is the expression for displacement.
- This expression is same as displacement equation which is used in oscillatory motion.
- As cosine function;**$y(x,t)= B \cos(kx – \omega t + \varphi)$,**As both sine and cosine function)**$y (x, t) = A \sin (kx – \omega t + \varphi) + B \cos(kx – \omega t + \varphi)$**

Mathematically:

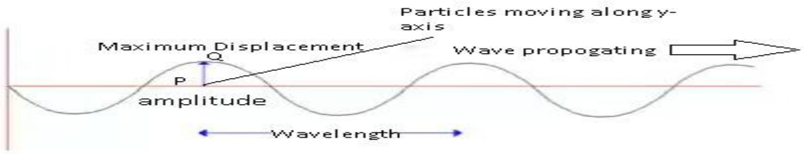
- Wave travelling along +X-axis: $y(x, t) = a \sin (kx – \omega t + \varphi)$.
- Consider $y=a\sin(kx – \omega t + \varphi)$=> $y/a=\sin(kx – \omega t + \varphi)$
- $\sin^{-1}(y/a) = kx-\omega t$ =>$kx=\sin^{-1}(y/a) +\omega t$
- **$x=(1/k)\sin^{-1}(y/a)+ (\omega t/k)$**
- Wave travelling along –X-axis: **$x=(1/k)\sin^{-1}(y/a)-(\omega t/k)$**(only change in the sign ofωt)
- Conclusion:-
- As time t increases the value of x increases. This implies the x moves along x-axis.
- As time t decreases the value of x decrease. This implies the x moves along (-)ive x-axis.

Amplitude and Phase of a wave

- Amplitude and phase together describes the position of the particle.
- Amplitude is the maximum displacement of the elements of the medium from their equilibrium positions as wave passes through them.
- It is denoted by A.

In case of transverse wave ,

- The distance between the point P and Q (in the Figure) is maximum displacement. This maximum displacement of the particles is known as amplitude.



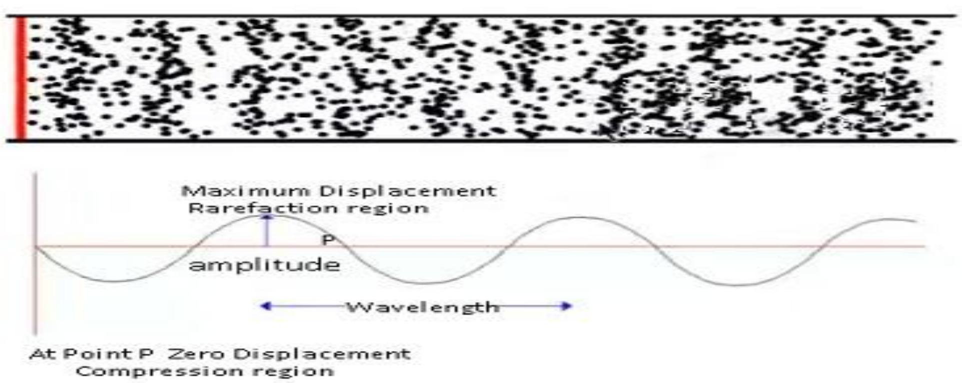
In case of longitudinal wave,

- There are regions of compressions (particles are closely packed) and rarefactions (particles are far apart).
- In compressions density of the wave medium is highest and in rarefactions density of the wave is lowest.
- Consider when the particle is at rarefaction, in that region as particle gets more space as a result the particles oscillates to the maximum displacement.
- Whereas in compressed region the particles oscillates very less as the space is not very much.
- The peak or the maximum amplitude is the centre of two compressed regions.Because at the centre of the two compressed region the particle is most free to displace to maximum displaced position.

Conclusion:-

- In case of longitudinal wave the particles will not oscillate to a very large distance. This displacement won't represent the amplitude as it is not maximum possible displacement.

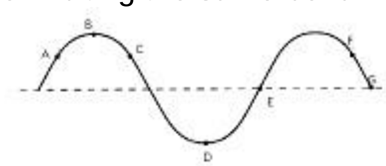
Amplitude is represented basically by the centre of the rarefaction region where the particle is most free to oscillate to its maximum displacement.



Phase

Phase of a wave describes the state of motion as the wave sweeps through an element at a particular position.

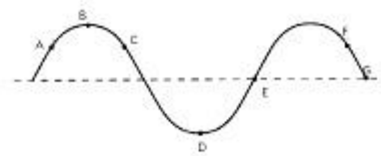
In-phase– Two points are said to be in-phase with each other when these two points are at the same position and they both are doing the same thing i.e. both the two points are exhibiting the same behaviour.



Points C and F are in phase with each other.

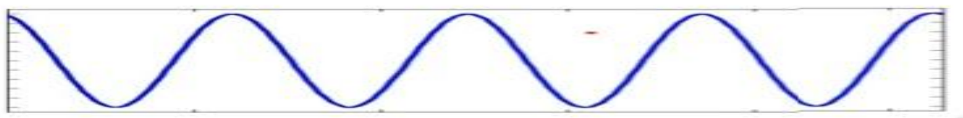
Out-of-phase –

- Two points are said to be out of phase even though they are at the same points but they are doing opposite thing i.e. both the points are exhibiting the different behaviour.
- Out of phase means which is not in phase.

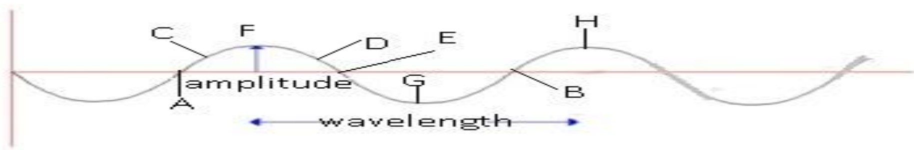


Points B and D,E and G are out of phase by 180^0

- Two waves can be completely in-phase or out of phase with each other. They can be partially in phase or out of phase with each other.
- Observations made from the figure (1).



Figure(1)



- Consider two points A and B on a wave.Their positions as well as their behaviour are same. Therefore points A and B are in phase.
- Consider points A and C on a wave. They are not in phase with each other as their position is not same.
- Similarly the points C and D are not in phase with each other as their positions are same but the behaviour is different. Thereforethey are not in phase with each other.
- Consider the points F and G their positions are same but the behaviour is totally opposite. So F and G are out of phase.
- Consider the points F and H;they are in phase with each other as their position is same as well as their behaviour.

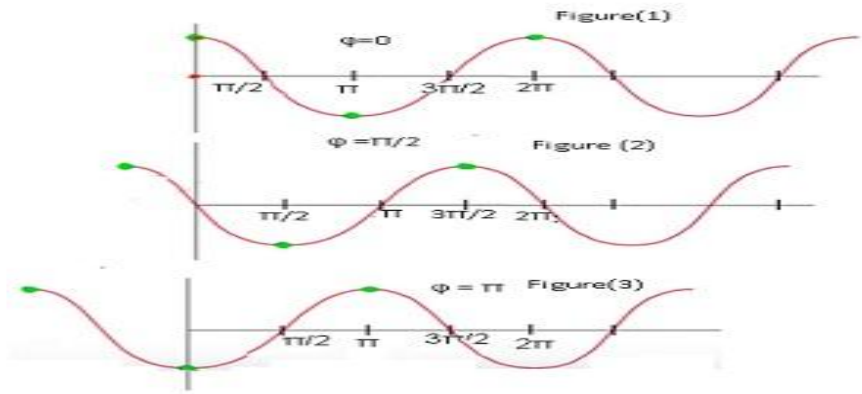
Expression of Phase

- y(x,t)= asin(kx – ωt + φ) where
- a= amplitude, (kx – ωt + φ)=phase

1. k=wavenumber,
2. x=displacement of wave along x-direction,
3. ω=angular frequency
4. t=time
5. φ=Initial phase angle.

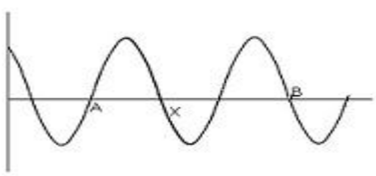
<u>Initial Phase angle</u>

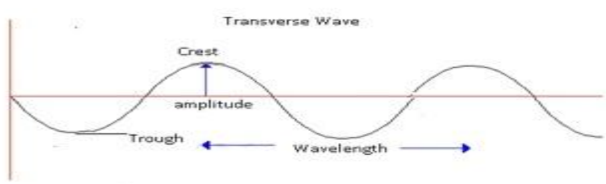
- Initial phase angle is defined as phase under initial conditions(t=0).
- Denoted by φ.
- It determines the angle between the starting position and the original.
- Consider the figure (1) the starting point is 0.i.e.φ=0.
- In figure (2) as it is not starting from 0, therefore φ=π/2.
- Similarly in figure (3) φ=π.
- So the initial phase angle determines from where the observation starts.
- Taking φ=0 does not affect any nature or properties of the wave.



Wavelength

- The term wavelength means length of the wave.
- Wavelength is defined as the minimum distance between two consecutive points in the same phase of wave motion.
- It is denoted by λ.
- In case of transverse wave we use the term **crest** for thepeak of the maximum displacement.
- The point of minimum displacement is known as **trough**.
- In case of transverse wave wavelength is the distance between two consecutive crests or distance between two consecutive troughs.
- In case of longitudinal wave wavelength is the distance between the two compressions or the distance between the two rarefactions provided the compressions or rarefactions are nearest.





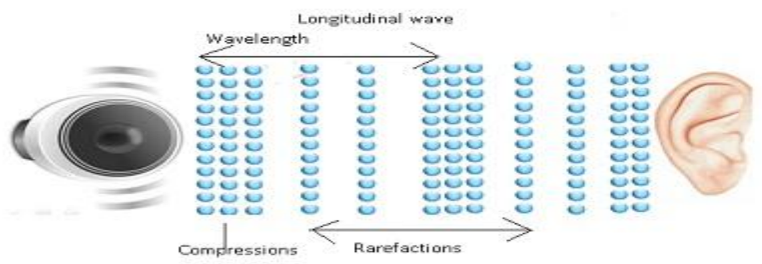
Problem:- A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, whatis the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340m s^{-1} and in water 1486 m s^{-1}.
Answer:- Frequency of the ultrasonic sound, v = 1000 kHz = 10^6 Hz
Speed of sound in air, v_a = 340 m/s
The wavelength (λ_r) of the reflected sound is given by the relation:
$\lambda_r = v_a / v = 340/10^6 = 3.4 \times 10^{-4}$ m
Frequency of the ultrasonic sound, v = 1000 kHz = 10^6 Hz
Speed of sound in water, v_w = 1486 m/s
The wavelength of the transmitted sound is given as:

$\lambda_r = 1486/10^6 = 1.49 \times 10^{-3}$ m

Problem:- A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of soundin the tissue in which the speed of sound is 1.7 km s^{-1}? The operating frequency of the scanner is4.2 MHz
Answer: Speed of sound in the tissue, v = 1.7 km/s = 1.7 × 10^3 m/s.
Operatingfrequency of the scanner, v = 4.2 MHz = 4.2 × 10^6 Hz.
The wavelength ofsound in the tissue is given as:

λ=v/v

= $(1.7x10^3)$/ $(4.2x10^6)$
=$4.1x10^{-4}$



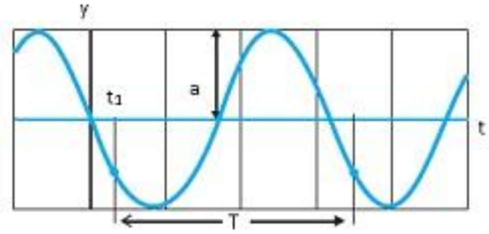
Wave Number
Wave number describes the number of wavelengths per unit distance.
Denoted by 'k'.
y(x,t)= a sin(kx – ωt + φ) assuming φ=0.

- At initial time t=0:-
  - y(x,0)=asin kx (i)
  - When x=x+λ then y(x+λ,0)=a sink(x+λ) (ii)
  - When x=x+2λ then y(x+2λ,0)=a sink(x+2λ)

Value of y is equal at all points because all the points' λ, 2λ are in phase with each other. Therefore,
From(i) and (ii) asin kx= a sink(x+λ) =asin(kx+k λ)
This is true if and only if: -k λ =2 πn, where n=1, 2, 3…
k=(2 π n)/ λ. This is the expression for wave number.
k is also known as **propagation constant**because it tells about the propagationof the wave.
Wave number is an indirect way of describing the propagation of wave.

Time Period, Frequency and Angular frequency
1. Time Period of a wave: -
1. Time Period of a wave is the time taken through one complete oscillation. It is denoted by'T'.
2. Frequency of a wave:-
1. Frequency of a wave is defined as number of oscillations per unit time.It is denoted by v.
2. v=1/T.
3. Angular frequency: -
1. Angular frequency is defined as the frequency of the wave in terms of a circular motion.
2. The term angular frequency is used only when there is an angle involved in the motion in that particular motion.
3. It is denoted by 'ω'.
4. For example:-In linear motion angular frequency is not used but in case of wave the term angular frequency is used.

- Relation of with ω frequency ν is given **$\omega = 2\pi\nu$ or $\omega = 2\pi/T$**.



A graph of the displacement of the stringelement at x = 0 as a function of time, as the sinusoidal wave passesthrough it. The amplitude a is indicated. Atypical period T, measured from anarbitrary time t_1 is also indicated.

Problem:- A wave travelling along astring is described by,$y(x, t) = 0.005 \sin (80.0 x – 3.0 t)$, in which the numerical constants are inSI units (0.005 m, 80.0 rad m^{-1}, and 3.0 rad s^{-1}). Calculate (a) the amplitude,(b) the wavelength, and (c) the period and frequency of the wave. Also, calculate thedisplacement y of the wave at a distance x = 30.0 cm and time t = 20 s?

Answer:-Applying displacementequation,

$y (x, t) = a \sin (kx – \omega t)$,

we find(a) the amplitude of the wave is 0.005 m = 5 mm.(b) the angular wave number k and angularfrequency ω arek = 80.0 m^{-1} and ω = 3.0 s^{-1}

$\lambda = 2\pi/k = (2 \pi)/ (80.0)m^{-1}$

= 7.85 cm

(c) Now we relate T to ω by the relation

$T = 2\pi/\omega = (2 \pi)/ (3.0) s^{-1}$

Frequency, $\nu = 1/T = 0.48$ Hz.

The displacement y at x = 30.0 cm andtime t = 20 s is given by

$y = (0.005 \text{ m}) \sin (80.0 \times 0.3 – 3.0 \times 20)$

$= (0.005 \text{ m}) \sin (–36 + 12\pi)$

$= (0.005 \text{ m}) \sin (1.699)$

$= (0.005 \text{ m}) \sin (97^0)$

~ 5 mm

Problem:-

A transverse harmonic wave on a string is described by $y(x,t) =3.0\sin(36t+0.081x+(\pi/4))$ Where x and y are in cm and t in s. The positive direction of x is from left to right. Is this a travelling wave or a stationary wave?

- If it is travelling, what are the speed and direction of its propagation?
- What are its amplitude and frequency?
- What is the initial phase at the origin?
- What is the least distance between two successive crests in the wave?

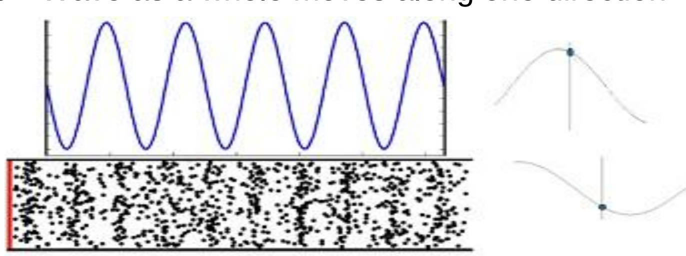
Answer:

- Yes; Speed = 20 m/s, Direction = Right to left
- 3 cm; 5.73 Hz,
- (π/4)
- 49 m

Explanation: The equation of a progressive wave travelling from right to left is given by the displacement
Function:y (x, t) = a sin (ωt + kx + Φ) … (i). The given equation is:
y(x, t) =3.0sin (36t+0.081x+(π/4))..(ii)
On comparing both the equations, we find that equation (ii) represents a travelling wave, propagating fromright to left.
Now, using equations (i) and (ii), we can write:
ω = 36 rad/s and k = 0.018 m^{-1}. We knowthat:
v=ω/ (2π) and λ= (2π)/k
Also, v = vλ
Therefore, v=(ω/ (2π)) x (2π/k) = ω/k
=36/ (0.018)=2000cm/s=20m/s
Hence, the speed of the given travelling wave is 20 m/s.
Amplitude of the given wave, a = 3 cm
Frequency of thegiven wave:
v=ω/ (2π) = 36/ (2x3.14) =5.73Hz
On comparing equations (i) and (ii), we find that the initial phase angle, Φ= (π/4)
The distance between two successive crests or troughs is equal to the wavelength of the wave.
Wavelength is given by the relation:
k=(2π)/λ
Therefore,
λ= (2π)/k = (2x3.14)/ (0.018) = 348.89cm=3.49m

Travelling Waves

- Travelling waves are those waves which travel from one medium to another.
- They are also known as progressive wave. Because they progress from one point to another.
- Both longitudinal and transverse waves can be travelling wave.
- Wave as a whole moves along one direction.



The speed of a travelling wave

- To determine the speed of a travelling wave.
- The propagation of the wave always takes place along the +(ive) x-axis.
- It is denoted by v.
- Consider transverse wave moving along (+) ive x axis.

- Displacement equation y (x, t) = a sin (kx – ωt) assuming φ=0.

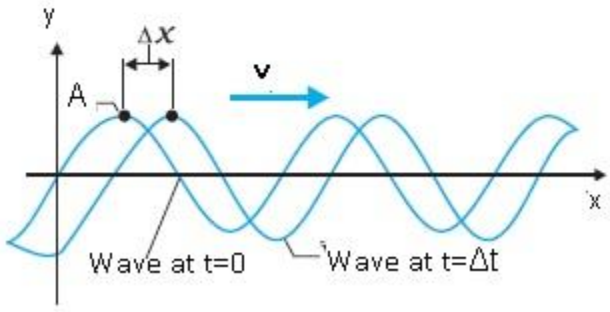
- In case of **transverse wave** the y denotes the movement of particles along y-axis.x denotes the direction of wave propagation in terms of x axis.
- y-axis denotes the displacement of the particles of the medium in case of transverse wave and in case of longitudinal wavex denotes the movement of the particles along x-axis.
- In case of **longitudinal wave** the motion of particles takes place along x-axis and not along y-axis.
- The form of equation remains the same for both transverse and longitudinal wave.
- Only the direction of propagation i.e. the direction of the movement of the particles differs in case of transverse and longitudinal waves.
- We have considered only the motion along the horizontal direction i.e. x-axis. i.e. motion of all the peaks and which is a motion of a straight line.
- Displacement takes place along the straight line along x-axis.
- y component does not play much role as it talks about the movement of particles of the medium.
- When we are considering the peaks as a result phase does not have any role. That is phase is constant.
- $y/a=\sin(kx - \omega t) \Rightarrow \sin^{-1}(y/a) = (kx - \omega t)$
  - $\Rightarrow x=(1/k)\sin^{-1}(y/a) + \omega t/k$ (where $(1/k)\sin^{-1}(y/a)$ is constant)
  - $\Rightarrow$ Speed $V=dx/dt = \omega/k$

(By differentiating with time,$(1/k)\sin^{-1}(y/a) = 0$ as differentiating a constant term is 0.)

- Therefore Wave Speed **V= ω/k**whereω=angular frequency and k=wave number.

<u>Different Expressions for Wave Speed</u>

- Wave speed V= ω/k. By definition of ω= (2 π)/T where T is period of the wave.
- Also k=(2 π)/λ .Therefore v=(2 π)/(T)/(2 π)/(λ) =λ/T
- **v= λv**
  - As wavelength of a wave increases as a result frequency of the wave decreases as a result speed of wave is constant.
  - Wave speed is determined by the properties of the medium.



The plots of y (x, t) = a sin (kx – ωt + φ) at two instants oftime differing by an interval Δt, at t = 0 and then at t = Δt. As the wave moves tothe right at velocity v, the entire curve shifts a distance Δx during Δt. The point A ridesthe waveform but the string element moves only up and down.

Speed of a transverse wave in a stretched string

- Consider a stretched string and if given transverse disturbance on one end then the disturbance travels throughout the string.
- Thereby giving rise to transverse waves.
- The particles move up and down and the waves travel perpendicular to the oscillation of the particles.
- Transverse wave speed determined by:
  - Mass per unit length- As mass gives rise to Kinetic energy.If no mass then no kinetic energy.Then there will be no velocity.
    - It is denoted by μ.
  - Tension-Tension is the key factor which makes the disturbance propagates along the string.
    - Because of tension the disturbance travels throughout the wave.
    - It is denoted by T.

<u>Dimensional Analysis to show how the speed is related to mass per unit length and Tension</u>

- $\mu = [M]/[L] = [ML^{-1}]$ (i)
- $T=F=ma =[M][LT^{-2}] = [MLT^{-2}]$ (ii)
- Dividing equation (i) by (ii) :- $[ML^{-1}]/[MLT^{-2}] = L^{-2}T^{-2} =[T/L]^2 =[TL^{-1}]^2 =1/v^2$
- Therefore $\mu/T = 1/v^2$
- **$v=C\sqrt{T}/ \mu$** where C=dimensionless constant
- Conclusion: v depends on properties of the medium and not on frequency of the wave.

<u>Problem:-</u> A steel wire 0.72 m long hasa mass of 5.0×10^{-3} kg. If the wire is under a tension of 60 N, what is the speed oftransverse waves on the wire?
<u>Answer</u>: -Mass per unit length of the wire,0.72 m
$\mu = 5.0\times10^{-3}$ kg
$= 6.9 \times10^{-3}$ kg m^{-1}
Tension, T = 60 N

The speed of wave on the wire is given by,$v=C\sqrt{T}/ \mu$

$\sqrt{(60)}/ (6.9x10^{-3}kgm^{-1}) = 93ms^{-1}$
<u>Problem:-</u> A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?
<u>Answer:-</u>
Mass of the string, M = 2.50 kg
Tension in the string, T = 200 N
Length of the string, l = 20.0 m
Mass per unit length,$\mu=M/l=2.50/20=0.125kgm^{-1}$
The velocity (v) of the transverse wave in the string is given by the relation: $v=\sqrt{T/\mu}$
$=\sqrt{200/0.125}=\sqrt{1600}=40m/s$
Therefore, time taken by the disturbance to reach the other end, t =l/v=20/40=0.50s

Problem:- A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20 °C = 343 m s^{-1}.

Answer:-

Length of the steel wire, l = 12 m

Mass of the steel wire, m = 2.10 kg

Velocity of the transverse wave, v = 343 m/s

Mass per unit length,μ=m/l= 2.10/12 =0.175kgm^{-1}

For tension T, velocity of the transverse wave can be obtained using the relation:

v= √T/μ

Therefore $T=v^2\mu$

$= (343)^2 \times 0.175 = 20588.575 \approx 2.06 \times 10^4$ N

Speed of a longitudinal wave in a stretched string

Longitudinal wave speed determined by:

- **Density** – Longitudinal wave is formed due to compressions (particles very close to each other) and rarefactions (particles are far from each other).
- At certain places it is very dense and at certain places it is very less dense.So density plays a very important role.
- It is denoted by ρ.
- **Bulk modulus**– Bulk modulus tells how does the volume of a medium changes when the pressure on it changes.
  - If we change the pressure of compressions orrarefactions then the volume of the medium changes.
  - It is denoted by B.

Dimensional Analysis to show how the speed is related to density and bulk modulus

- ρ =mass/volume= $[ML^{-3}]$
- B = - (Change in pressure (ΔP))/Change in volume(ΔV/V))
- ΔV/V is a dimensionless quantity as they are 2 similar quantities.
- ΔP =F/A = ma/A =$[M][LT^{-2}]/[L^2] = [ML^{-1}T^{-1}]$
- Dividing ρ/B= $[ML^{-3}]/[ML^{-1}T^{-2}] = [L^{-2}T^2]=[TL^{-1}]^2 =1/v^2$
- Therefore ρ/B = $1/v^2$ or v^2=B/ ρ
- **v= C √ (B/ ρ)** where C=dimensionless constant
- In case of fluids :- v= C √ (B/ ρ)
- In case of solids :- v= C √ (Y/ ρ)

Problem:- Estimate the speed ofsound in air at standard temperature andpressure. The mass of 1 mole of air is29.0 $\times 10^{-3}$ kg.

Answer:-

We know that 1 mole of any gasoccupies 22.4 litres at STP. Therefore, density of air at STP is:ρ_o = (mass of one mole of air)/ (volume of onemole of air at STP)

29.0 x 10^3 kg/22.4x10^3 m^3

= 1.29 kg m^{-3}

According to Newton's formula for the speedof sound in a medium, we get for the speed of

sound in air at STP,

v = $[(1.01\text{x}10^5 Nm^{-2})/ (1.29kgm^{-3})]^{1/2}$

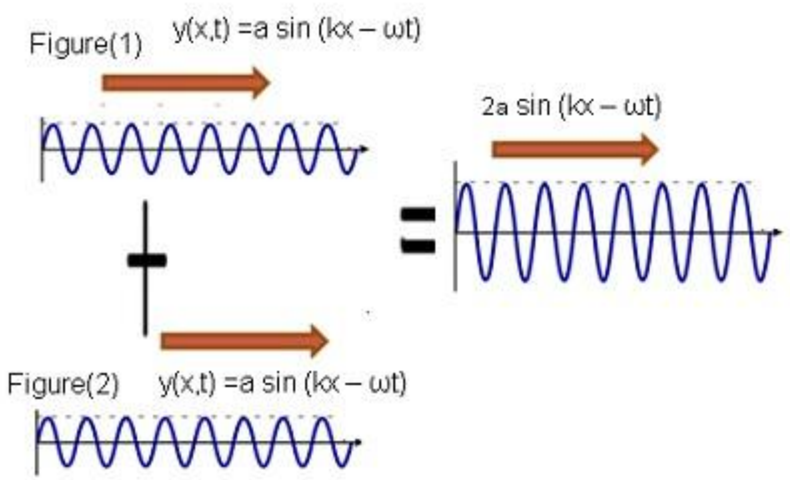
=280ms^{-1}

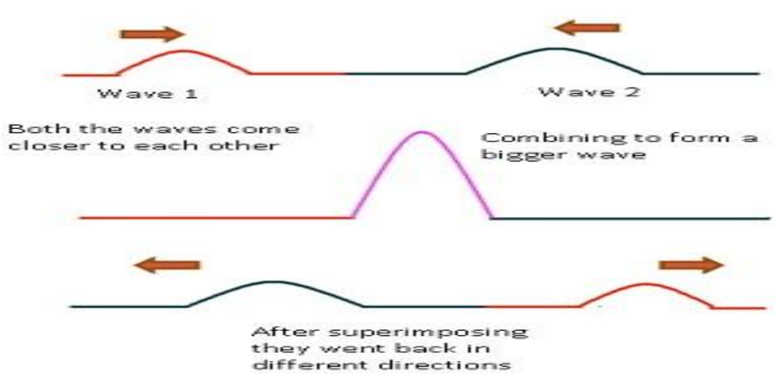
The principle of superposition of waves

- Principle of superposition of waves describes how the individual waveforms can be algebraically added to determine the net waveform.
- Waveform tells about the overall motion of the wave.It does not tell about individual particles of the wave.
- Suppose we have 2 waves and
- Example of superposition of waves is Reflection of waves.
- Mathematically: -

Case1:-

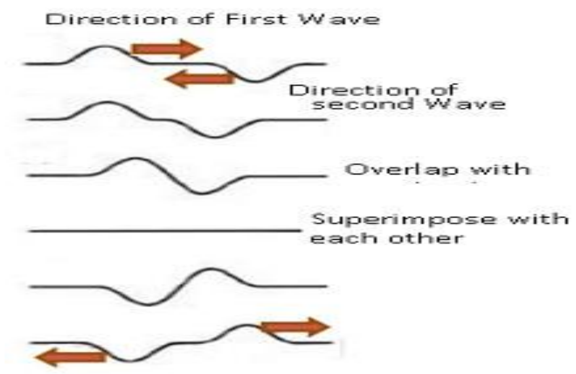
- Consider 2waves which are in phase with each other. They have the same amplitude, same angular frequency, and same angular wave number.
- If wave 1 is represented by $y_1(x, t) = a \sin (kx - \omega t)$.
- Wave 2 is also represented by $y_2(x, t) = a \sin (kx - \omega t)$.
- By the principle of superposition the resultant wave ($2a \sin (kx - \omega t)$) will also be in phase with both the individual waves but the amplitude of the resultant wave will be more.

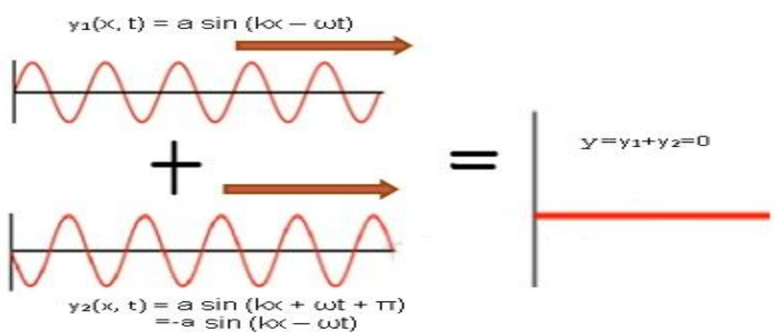




Case2:-

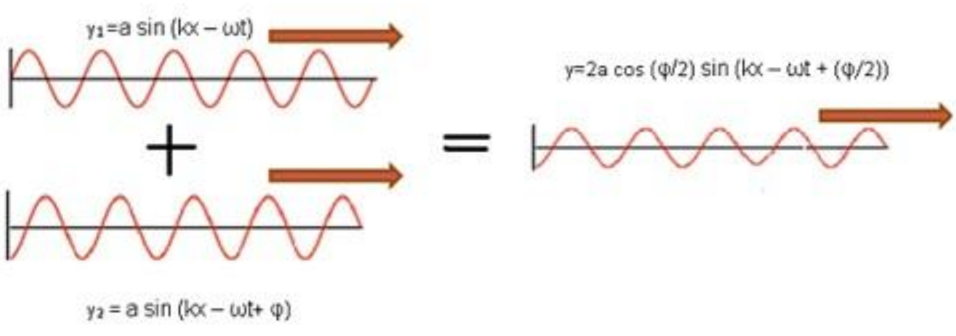
- Consider when the two waves are completely out of phase.i.e. $\varphi = \pi$
- If wave 1 is represented by $y_1(x, t) = a \sin (kx - \omega t)$.
- Wave 2 is represented by $y_2(x, t) = a \sin (kx - \omega t + \pi)$.
- $\Rightarrow y_2 = a\sin(\pi - (-kx + \omega t)) \Rightarrow y_2 = -a \sin(kx - \omega t)$
- Therefore by superposition principle $y = y_1 + y_2 = 0$





Case 3:-

- Consider when the two waves partially out of phase $\varphi>0$; $\varphi<\pi$
- If wave 1 is represented by $y_1(x, t) = a \sin(kx - \omega t)$.
- Wave 2 is represented by $y_2(x, t) = a \sin(kx - \omega t + \varphi)$.
- Therefore by the principle of superposition of waves , $y = y_1 + y_2$
- $= a[\sin(kx - \omega t) + \sin(kx - \omega t + \varphi)]$
- $\mathbf{y = 2a \cos(\varphi/2) \sin(kx - \omega t + (\varphi/2))}$
    - (By using the formula sinA +sinB=2sin(A+B)/2) cos(A-B)/2)
- Amplitude = $2a \cos(\varphi/2)$ and Phase will be determined by $(\varphi/2)$.



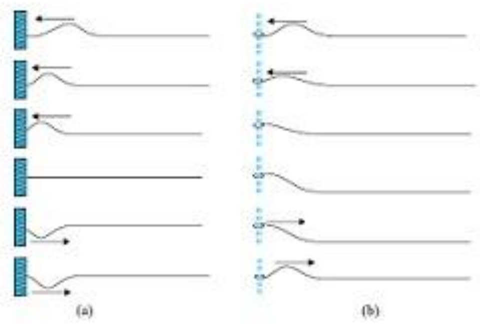
Reflection of waves

Reflection of waves is the change in the direction of a wave upon striking the interface between two materials.

When a wave strikes any interface between any two mediums the bouncing back of wave is termed as reflection of waves.

Interface can be categorised into 2 types:

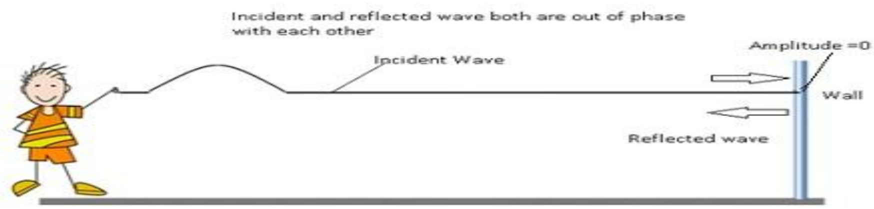
- **Open boundary**: - When a wave strikes an interface in case of open boundary it will get reflected as well as refracted.
- **Closed boundary** or a rigid boundary: - When a wave is incident on an interface it will completely get reflected. Example:-Wave striking wall(echo)

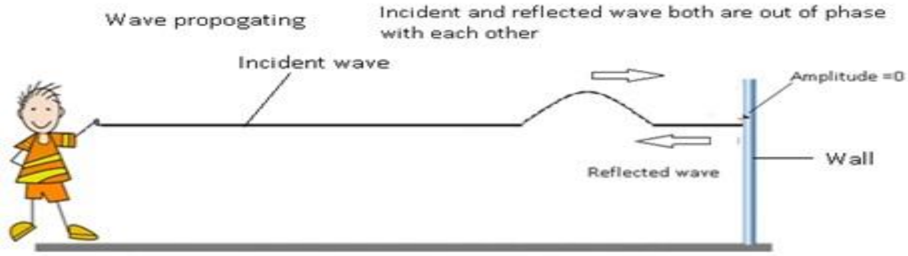


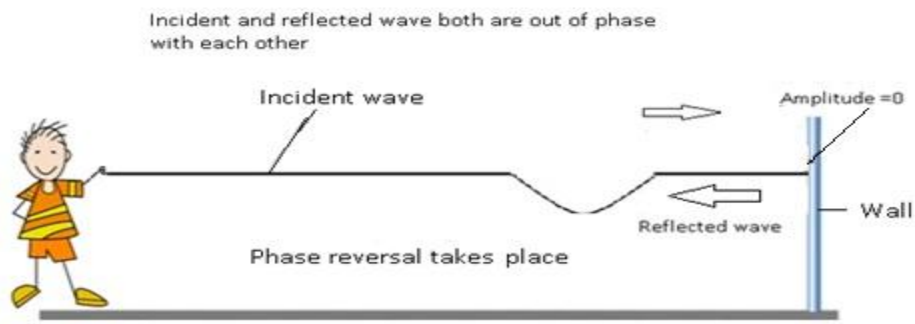
(a) A pulse incident from the right is reflected at the left end ofthe string, which is tied to a wall. Note that the reflectedpulse is inverted from the incident pulse. (b) Here the left end is tied to a ring that can slide up and down without friction onthe rod. Now the reflected pulse is not inverted by reflection.

<u>Reflection at rigid boundary</u>

- Consider a string which is fixed to the wall at one end. When an incident wave hits a wall,it will exerta force on the wall.
- By Newton's third law, the wall exerts an equal and opposite force of equal magnitude on the string.
- Since the wall is rigid wall won't move, therefore no wave is generated at the boundary.This implies the amplitude at the boundary is 0.
- As both the reflected wave and incident wave are completely out of phase at the boundary.Therefore $\varphi=\pi$.
- Therefore, $y_i(x, t) = a \sin (kx - \omega t)$,
- $y_r(x, t) = a \sin (kx + \omega t + \pi) = - a \sin (kx + \omega t)$
- By superposition principle $y= y_i + y_r =0$
- **<u>Conclusion</u>**: -
  - The reflection at the rigid body will take place with a phase reversal of π or 180.

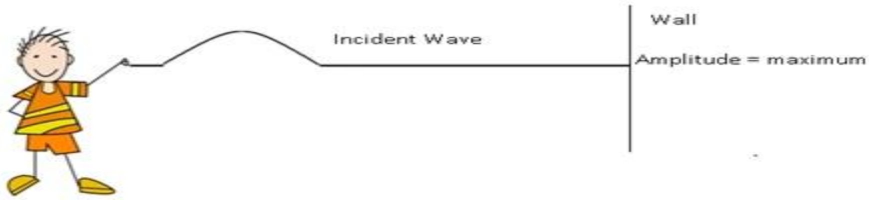






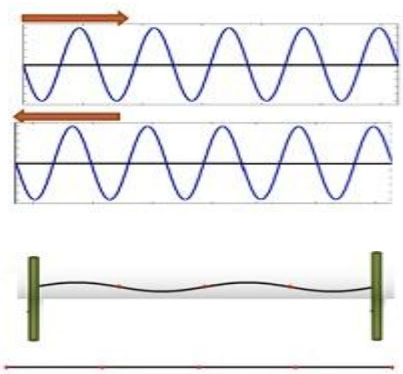
<u>Reflection at open boundary</u>

- The reflection at an open boundary will take place without any phase change.
- In this case at boundary pulse is generated.Therefore amplitude at the boundary is maximum.
- This means the reflected wave and incident wave are in phase with each other.As a result the phase difference φ=0.
- Therefore, $y_i(x, t) = a \sin(kx - \omega t)$,
- $y_r(x, t) = a \sin(kx - \omega t)$.By superposition principle $y= y_i + y_r =2a \sin(kx - \omega t)$



Standing (Stationary) Waves

- A stationary wave is a wave which is not moving,i.e. it is at rest.
- When two waves with the same frequency,wavelengthand amplitude travelling in opposite directions will interfere they produce a standing wave.

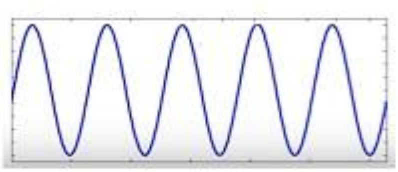


- Conditions to have a standing wave:- Two travelling waves can produce a standing wave, if the waves are moving in opposite directions and they have the same amplitude and frequency.
- At certain instances when the peaksof both the waves will overlap. Then both the peaks will add up to form the resultant wave.
- At certain instances when the peak of the one wave combine with the negative of the second wave .Then the net amplitude will become 0.
- As a result a standing wave is produced. In case of stationary wave the waveformdoes not move.
- Explanation:-

- Consider I[st] wave in the figure and suppose we have a rigid wall which does not move. When anincident wave hits the rigid wall it reflects back with a phase difference of π.
- Consider II[nd] wave in the figure, when the reflected wave travels towards the left there is another incident wave which is coming towards right.
- The incident wave is continuously coming come from left to right and the reflected wave will keep continuing from right to left.
- At some instant of time there will be two waves one going towards right and one going towards left as a result these two waves will overlap and form a standing wave.

- Mathematically:
- Wave travelling towards left $y_l(x,t) = a \sin(kx - \omega t)$ and towards right $y_r(x,t) = a \sin(kx + \omega t)$
- The principle of superposition gives, for the combined wave
- $y(x, t) = y_l(x, t) + y_r(x, t) = a \sin(kx - \omega t) + a \sin(kx + \omega t)$
- **$y(x,t) = (2a \sin kx) \cos \omega t$** (By calculating and simplifying)
- The above equation represents the standing wave expression.
- Amplitude = $2a \sin kx$.
  - The amplitude is dependent on the position of the particle.
  - The $\cos \omega t$ represents the time dependent variation or the phase of the standing wave.

Difference between the travelling wave and stationary wave

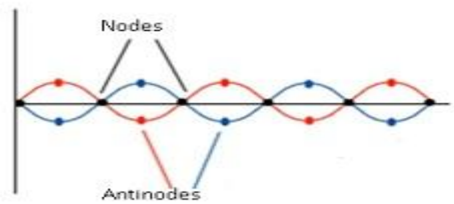
Travelling Wave(Progressive Wave)	Stationary Wave (Standing wave)
Waveform moves. Movement of the waveform is always indicated by the movement of the peaks of the wave.	Waveform doesn't move. Peaks don't move.
Wave amplitude is same for all the elements in the medium. Denoted by 'A'.	Wave amplitude is different for different elements. Denoted by asinkx.
Amplitude is not dependent on the position of the elements of the medium.	Amplitude is dependent on the position of the elements of the medium.
$y(x,t)=a\sin(kx - \omega t + \varphi)$	$y(x,t)=2a\sin(kx)\cos(\omega t)$
	

Nodes and Antinodes of Standing Wave

- The amplitude of a standing wave doesn't remain the same throughout the wave.
- It keeps on changing as it is a function of x.
- At certain positions the value of amplitude is maximum and at certain positions the value of amplitude is 0.
- **Nodes**: - Nodes represent the positions of zero amplitude.
- **Antinodes**: - Antinodes represent the positions of maximum amplitude.

<u>Nodes:-</u>

- At nodes, amplitude is 0.
- In case of the standing wave amplitude is given as :- 2asinkx
- => 2asinkx = 0 ,=>sinkx = 0,=>sinkx =sin n π => kx=n π
- The value of x represents position of nodes where amplitude is 0.
- x=(nπ)/k … equation(i)
- From the definition of k=(2π)/λ ... equation(ii)
- The position of nodes is represented by: -**x=(n λ)/2**from(i) and (ii),where n=1, 2, 3…
- **Note:** -Half a wavelength (λ/2) separates two consecutive nodes**.**

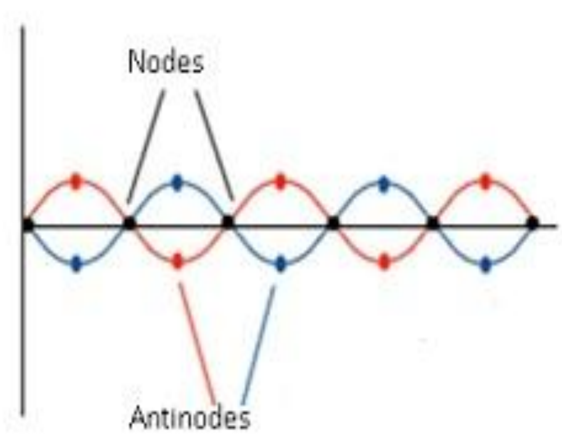


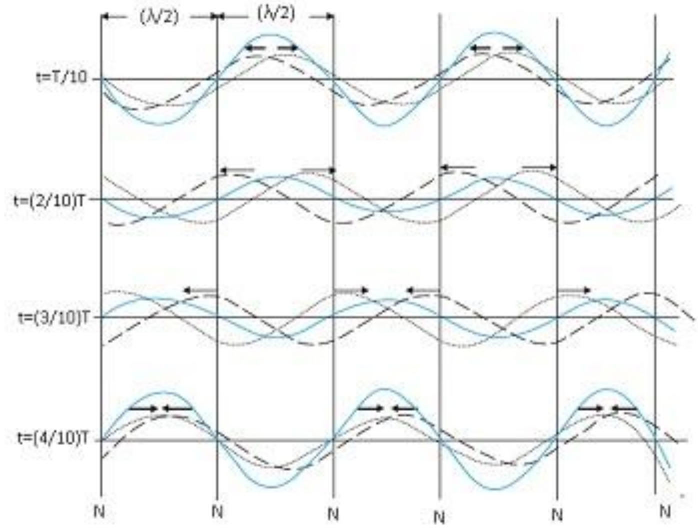
<u>Antinodes</u>:-

- At antinodes, amplitude is maximum.
- In case of the standing wave amplitude is given as :- 2asinkx
- => 2asinkx = maximum. This value is maximum only when sinkx=1.
- => sinkx = sin(n+(1/2)) =>π=>kx=(n+(1/2)) =>π=>((2 π)/ λ) x= (n+(1/2)) π
- The position of nodes is represented by**:- x= (n+(1/2))(λ/2) ;** n=0,1,2,3,4 …

Note: -

1. Half awavelength separates two consecutive nodes.
2. Antinodes are located half way between pairs of nodes.





The formation of a standing wave in a stretched string. Two sinusoidal waves of same amplitudetravel along the string in opposite directions. The set of pictures represent the state of displacementsat four different times. The displacement at positions marked as N is zero at all times. These positionsare called nodes.

Nodes and Antinodes: system closed at both ends

- System closed at both ends means both the ends are rigid boundaries.
- Whenever there is rigid body there is no displacement at the boundary. This implies at boundary amplitude is always 0. Nodes are formed at boundary.
- Standing waves on a string of length L fixed at both ends have restricted wavelength.
- This means wave will vibrate for certain specific values of wavelength.
- At both ends,nodes will be formed.=>Amplitude=0.
- Expression for node x =(nλ)/2.This value is true when x is 0 and L.
- When x=L:- L=(nλ)/2 =>**λ=(2L)/n** ; n=1,2,3,4,…..
- λ cannot take any value but it can take values which satisfy λ=(2L)/n this expression.
- That is why we can say that the standing wave on a string which is tied on both ends has the restricted wavelength.
- As wavelength is restricted therefore wavenumber is also restricted.
- ν =v/λ (relation between wavelength and frequency)
- Corresponding frequencies which a standing wave can have is given as: -**ν= (vn)/2L**where v= speed of the travelling wave.
- These frequencies are known as **natural frequency** or **modes of oscillations**.

Modes of Oscillations:-

- **ν= (vn)/2L** where v=speed of the travelling wave, L=length of the string, n=any natural number.
- **First Harmonic:-**
  - For n=1, mode of oscillation is known as **Fundamental mode**.
  - Therefore ν_1=v/(2L).This is the lowest possible value of frequency.
  - Therefore $\nu_1$is the lowest possible mode of the frequency.
  - 2 nodes at the ends and 1 antinode.

- **Second Harmonic:-**
  - For n=2, v_2=(2v)/ (2L) =v/L
  - This is second harmonic mode of oscillation.
  - 3 nodes at the ends and 2 antinodes.
- **ThirdHarmonic:-**
  - For n=3,v_3 = (3v)/ (2L).
  - This is third harmonic mode of oscillation.
  - 4 nodes and 3 antinodes.

Problem:- Find the frequency of note emitted (fundamental note) by a string 1m long and stretched by a load of 20 kg, if this string weighs 4.9 g. Given, g = 980 cm s^{-2}?
Answer:-
L = 100 cm T = 20 kg = 20 × 1000 × 980 dyne

m= 4.9/100 = 0.049 g cm^{-1}
Now the frequency of fundamental note produced,

v= (1/2L) √ (T/m)

v = 1/ (2x100) √(20x1000x980)/(0.049)

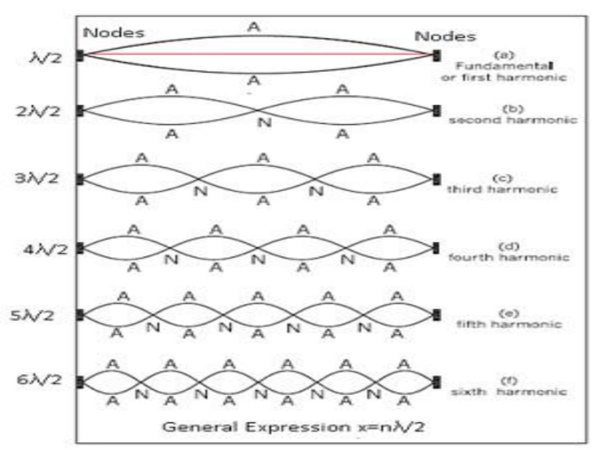
=100Hz

Problem:- A pipe 20 cm long is closed at one end, which harmonic mode of the pipeis resonantly excited by a 430 Hzsource? Will this same source can bein resonance with the pipe, if both ends are open? Speed of sound = 340ms^{-1}?
Answer:-
The frequency of nth mode of vibration of a pipe closed at one endis given by

v_n=(2n 1)v/4L
v=340ms^{-1}; L=20cm=0.2m;v_n=430Hz.
Therefore 430= ((2n-1) x 340)/ (4x0.2)

=>n=1

Therefore, first mode of vibration of the pipe is excited, for open pipe sincen must be an integer, the same source cannot be in resonance with thepipe with both ends open.



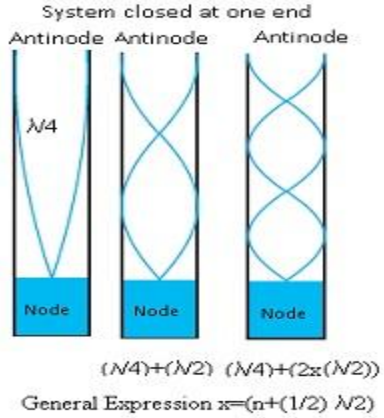
Stationary waves in a stretched string fixed at bothends. Various modes of vibration are shown.

Nodes and Antinodes: system closed at one end

- For a system which is closed at one end,only one node is formed at the closed end.
- Consider there is one fixed end(x=0) and one open end(x=L), and a string is attached between these two ends.
- At x=L, antinodes will be formed.This means amplitude will be maximum at this end.
- Condition for formation of antinodes is x= (n+ (1/2)) (λ/2).=>L=(n+ (1/2)) (λ/2)
- =>**λ=(2L)/(n+(1/2))**.This expression shows that the values of wavelength is restricted. n=0, 1, 2, 3…
- Corresponding frequencies will be v= v/(2L)(n+1/2) (By using v=v/λ);n=0,1,2,3, …

Modes of oscillations:-

- Fundamental frequency:- Also known as First Harmonic:-It corresponds to lowest possible value for n. That is n=0.
- The expression for fundamental frequency is v_0 = v/(2L)x1/2 = v/(4L)
- **Odd Harmonics**
  - n=1 ; v =v/(2L)(1+(1/2)) = (3v)/(4L) =3v_0
  - n=2; v =v/(2L)(2+(1/2))= (5v)/(4L)=5v_0
  - n=3;v =v/(2L)(3+(1/2))=(7v)/(4L)=7v_0
- For a system which is closed at one end and open at another end will get one fundamental frequency and all other odd harmonics.



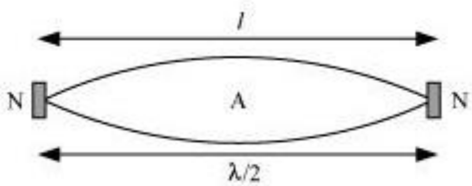
Problem:- A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel?

Answer:-

Length of the steel rod, l = 100 cm = 1 m

Fundamental frequency of vibration, v = 2.53 kHz = 2.53 × 10^3 Hz

When the rod is plucked at its middle, an antinode (A) is formed at its centre, and nodes (N) areformed at its two ends, as shown in the given figure.



The distance between two successive nodes is$\lambda/2$.

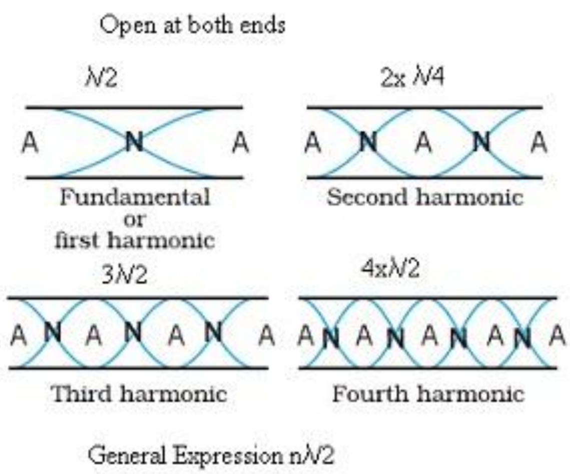
Therefore, $l=\lambda/2$

$\lambda = 2l = 2x1 = 2m$.

The speed of sound in steel is given by the relation:

$v = \nu\lambda$

$= 2.53 \times 10^3 \times 2$
$= 5.06 \times 10^3$ m/s
$= 5.06$ km/s



Problem:- A pipe, 30.0cm long, is open at both the ends. Which harmonic mode of the pipe resonates a 1.1 kHz source? Will resonance with the same source be observed if one end of the pipe is closed? Take the speed of the sound in air as $330ms^{-1}$?

Answer:-

1. FundamentalFrequency: $-\nu_1 = (V)/(2L)$
   Nth harmonic = ν_n=nV/(2L); n=1, 2, 3, 4 ….
   =(nx330)/(2x0.3) = nx550m/s.

   $\nu_n=1.1x10^3$Hz=1100Hz. (Source frequency given)
   Therefore nx550=1100 => n=2. It is the second harmonic.

2. Fundamental Frequency: - ν_1= (V)/ (4L)
   Odd harmonics :- (3V)/ (4L), (5V)/ (4L), (7V)/ (4L) ….

   =>$\nu_1=\nu_Y$ =275Hz. Where ν_Y = Fundamental Frequency
   =>$n\nu_1$ = 1100 => n=1100/275=4
   This should correspond to 4^{th} harmonic but 4^{th} harmonic cannot be present as only odd harmonics are present.
   Therefore the same source cannot be resonated if the pipe is closed at one end.

Beats

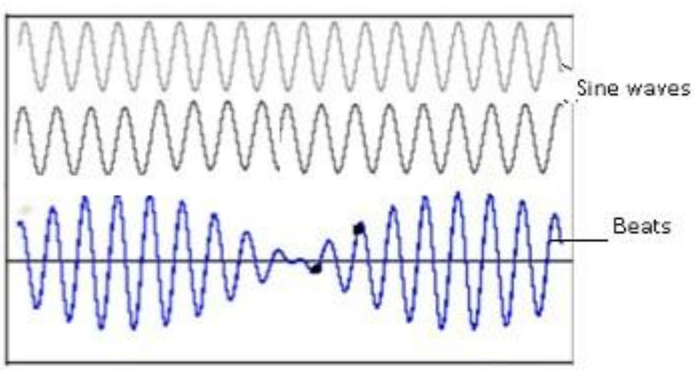
Beats is the phenomenon caused by two sound waves of nearly same frequencies and amplitudes travelling in the same direction.

For example:-

- Tuning of musical instruments like piano, harmonium etc. Before we start playing on these musical instruments they are set against the standard frequency. If it is not set a striking noise will keep on coming till it is set.

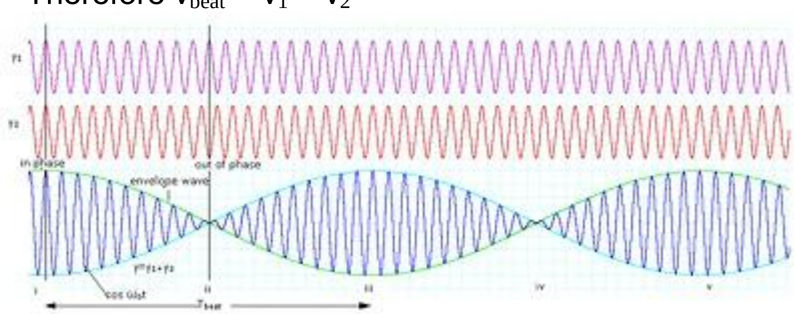
Mathematically

- Consider only the time dependent and not the position dependent part of the wave.
- s_1=a cos ω_1t and s_2=a cos ω_2t; where amplitude and phase of the waves are same,but the frequencies are varying. Also considering $\omega_1 > \omega_2$.
- When these 2 waves superimpose s= $s_1 + s_2$=a[cos ω_1t + cos ω_2t]
- By simplifying , 2a (cos(ω_1 - ω_2)/2)t cos(ω_1 + ω_2)/2)t)
- =>ω_1 - ω_2 is very small as $\omega_1 > \omega_2$.Let (ω_1 - ω_2)=ω_b
- =>ω_1 + ω_2 is very large. Let (ω_1 + ω_2)=ω_a
- **s= 2a cos ω_bt cos ω_at**
- cosω_at will vary rapidly with time and 2acosω_bt will change slowly with time.
- Therefore we can say 2acosω_bt = constant. As a result 2acosω_bt = amplitude as it has small angular variation.



<u>Beat Frequency</u>

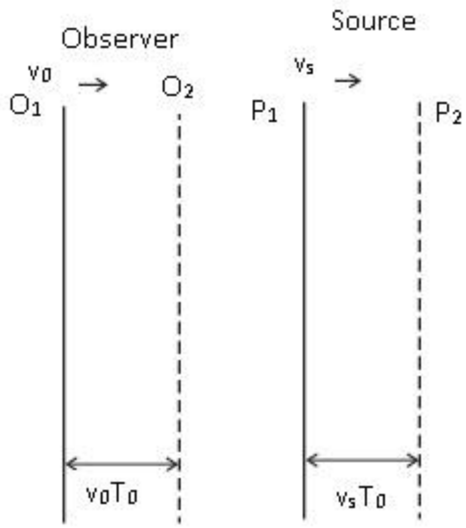
- Beat frequency can be defined as the difference in the frequencies of two waves.
- Consider if there is a wave of frequency ω_1 and another wave of frequency ω_2.Then the beat frequency will be ω_1- ω_2.
- It is denoted by ω
- Also ω = 2πv
- Therefore $v_{beat} = v_1 – v_2$



Problem:- Two sitar strings A and Bplaying the note 'Dha' are slightly out oftune and produce beats of frequency 5 Hz.The tension of the string B is slightlyincreased and the beat frequency is foundto decrease to 3 Hz. What is the originalfrequency of B if the frequency of A is 427 Hz?
Answer:- Increase in the tension of a stringincreases its frequency. If the original frequencyof B (v_B) were greater than that of A (v_A), furtherincrease in v_B should have resulted in anincrease in the beat frequency. But the beatfrequency is found to decrease. This shows that$v_B < v_A$. Since $v_A – v_B$ = 5 Hz, and v_A = 427 Hz, weget v_B = 422 Hz.

Doppler's Effect

- Doppler Effect is the phenomenon of motion-related frequency change.
- Consider if a truck is coming from very far off location as it approaches near our house, the sound increases and when it passes our house the sound will be maximum. And when it goes away from our house sound decreases.
- This effect is known as Doppler Effect.
- A person who is observing is known as **Observer** and object from where the sound wave is getting generated it is known as **Source**.
- When the observer and source come nearer to each other as a result waves get compressed. Therefore wavelength decreases and frequency increases.
- **Case 1**:- stationary observer and moving source
- Let the source is located at a distance L from the observer.
- At any time t_1, the source is at position P_1.
- Time taken by the wave to reach observer =L/v where v=speed of the sound wave.
- After some time source moves to position P_0 in time T_0.
- Distance between P_1 and P_0 =v_sT_o where v_s is the velocity of the source.
- Let $t_2$be the time taken by the second wave to reach the observer
  - Total time taken by the for the second wave to be sent to the observer = $T_o +(L+v_sT_o)/v$
  - Total time taken by the for the third wave to be sent to the observer=$2T_o +(L+2v_sT_o)/v$
  - Therefore for nth point $t_{n+1} =nT_o +(L+nv_sT_o)/v$
  - =>In time $t_{n+1}$the observer captures n waves.
- Total time taken by the waves to travel Time period T= $(t_{n+1} - t_1)/n$
- $=T_o +(v_sT_o)/v \Rightarrow T=T_o(1+v_s/v)$
- Or v= 1/T
- $\Rightarrow v = v_0(1+v_s/v)^{-1}$
- By using binomial Theorem,$\mathbf{v= v_0 (1- v_s/v)}$
- If the source is moving towards the observer the expression will become $\mathbf{v= v_0 (1+ v_s/v)}$
- **Case 2**:- moving observer and stationary source
- As the source is not moving therefore v_s is replaced by $-v_0$.
- Therefore $\mathbf{v= v_0 (1+ v_0/v)}$



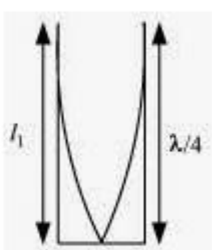
The observer O and the source S, bothmoving respectively with velocities v_0 and v_s . They are at position O_1 and P_1 at timet = 0, when the source emits the first crest of a sound, whose velocity is v withrespect to the medium. After one period, t = T_0, they have moved to O_2 and P_2,respectively through distances v_0 T_0 and v_s T_0, when the source emits the nextcrest.

Problem:- A metre-long tube opens at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

Answer:-

Frequency of the turning fork, v = 340 Hz

Since the given pipe is attached with a piston at one end, it will behave as a pipe with one end closedand the other end open, as shown in the given figure.



Such a system produces odd harmonics. The fundamental note in a closed pipe is given by therelation

$l_1=\lambda/4$ Where,

Length of the pipe,l_1=25.5cm=0.255m

Therefore,$\lambda=4l_1$ =4x0.255 = 1.02m

The speed of sound is given by the relation:

$v=v\lambda$ =340x1.02 =346.8m/s

Wisdom Education Academy Mob: 8750387081
Notes provided by

Mohd Sharif
B.Tech. (Mechanical Engineer)
Diploma (Mechanical Engineer)
J.E. in DSIIDC.
Trainer & Career Counsellor
(10 year experience in Teaching Field)
(3+ Year experience in Industrial Field)