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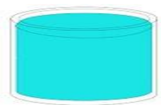
Introduction:Fluids

- Fluids can be defined as any substance which is capable of flowing.
- They don't have any shape of their own.
- For example:-water which does not have its own shape but it takes the shape of the container in which it is poured.

But when we pour water in a tumbler it takes the shape of the tumbler



- Both liquids and gases can be categorised as fluids as they are capable of flowing.
- Volume of solids,liquids and gas depends on the stress or pressure acting on it.
- In this chapter we will study if we apply force on the fluid how does it affects the internal properties of fluids.
- Fluids offer very little resistance to shear stress.
- We will also study some characteristic properties of fluids.

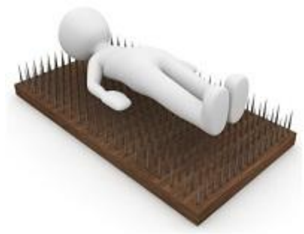
Pressure

- Pressure is defined as force per unit area.
- Pressure = Force/Area
- For Example:-
  - Consider a very sharp needle which has a small surface area and consider a pencil whose back is very bluntand has more surface area than the needle.
  - If we poke needle in our palm it will hurt as needle gets pierced inside our skin.Whereas if we poke the blunt side of the pencil into our hand it won't pain so much.
  - This is because area of contact between the palm and the needle is very small therefore the pressure is large.
  - Whereas the area of contact between the pencil and the palm is more therefore the pressure is less.

- Conclusion: Two factors which determine the magnitude of the pressure are:-
- Force – greater the force greater is the pressure and vice-versa.
- Coverage area –greater the area less is the pressure and vice-versa.

Example:-

- Consider a stuntman lying on the bed of nails which means there are large numbers of nails on any rectangular slab. All the nails are identical and equal in height.
- We can see that the man is not feeling any pain and he is lying comfortably on the bed. This is because there isa large number of nails and all the nails are closely spaced with each other.
- All the small pointed nails make large surface area therefore the weight of the body is compensated by the entire area of all the nails.
- The surface area increases therefore pressure is reduced.
  - But even if one nail is greater than the others then it will hurt. Because then the surface area will be less as a result pressure will be more.



Stuntman lying on bed of nails.

Problem:- A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

Answer:-

Mass of the girl, m = 50 kg

Diameter of the heel, d = 1 cm = 0.01 m

Radius of the heel, r = d/2 = 0.005m

Area of the heel = $\pi r^2 = \pi (0.005)^2 = 7.85 \times 10^{-5}\ m^2$

Force exerted by the heel on the floor:

F = mg = 50 × 9.8 = 490 N

Pressure exerted by the heel on the floor:

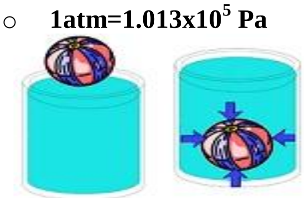
$P = Force/Area = 490/(7.85 \times 10^{-5})$

$= 6.24 \times 10^{6}\ N\ m^{-2}$

Therefore, the pressure exerted by the heel on the horizontal floor is $6.24 \times 10^{6}\ Nm^{-2}$.

Pressure in Fluids:-

- Normal force exerted by fluid per unit area.
- This means force is acting perpendicular to the surface of contact.
- Consider a body submerged in the water, force is exerted by the water perpendicular to the surface of the body.
  - If there is no force applied perpendicularly but in the parallel direction then there will be motion along the horizontal direction.
  - Since fluid is at rest and body is submerged in the fluid. Therefore there cannot be motion along the horizontal direction.
  - Therefore we always say the force is applied perpendicularly.
- Pressure is a scalar quantity. Because the force here is not a vector quantity but it is the component of force normal to the area.
- Dimensional Formula $[ML^{-1}T^{-2}]$
- I Unit: N/m^2 or Pascal(Pa).
- Atmosphere unit (atm) is defined as pressure exerted by the atmosphere at sea level.It is a common unit of pressure.
- **$1atm = 1.013 \times 10^5$ Pa**



Problem:- The two thigh bones (femurs), each of cross-sectional area10 cm^2 support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs.

Answer:-

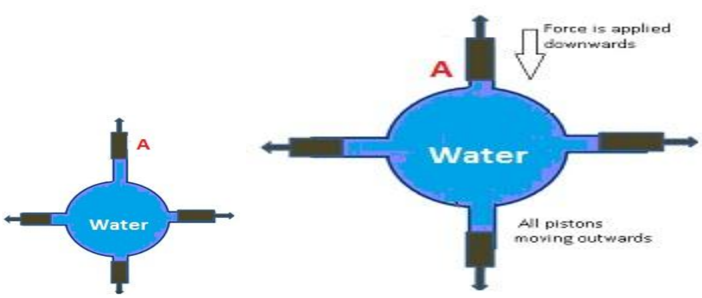
Total cross-sectional area of thefemurs is A = 2 × 10 cm^2 = $20 \times 10^{-4}\ m^2$. The force acting on them is F = 40 kg wt. = 400 N(taking g = 10 m s^{-2}). This force is acting vertically down and hence, normally on thefemurs.

Thus, the average pressure is= 2 × 105 N m^{-2}

Pascal's Law

- Pascal's law states that if the pressure is applied to uniform fluids that are confined,the fluids will then transmit the same pressure in all directions at the same rate.
- Pascal's law holds good only for uniform fluids.
- For example:-
  - Consider a vessel filled with water which is uniform throughout as there is only one type of fluid which is water.
  - Consider a vessel which has oil and water then it is not uniform.As it have two different fluids.

- Fluid should be confined meaning fluid is present within region in space.It is not allowed to spread.
- For example 1:-
  - A balloon filled with water and when we press it hard against the wall.
  - We will see the shape of the balloon changes. This is because if we apply force on balloon, pressure is exerted on the water.
  - Water is uniform fluid and it is confined with in this balloon and is not allowed to spread.
  - On applying pressure it is transmitted in all other directions.
- For example 2:-
  - Consider a vessel of circular shape filled with water which has 4 openings and in the entire openings 4 pistons are attached.
  - Apply force on the first piston; this piston will move inward and all other pistons will move outwards.
  - This happens because when this piston moves inwards the pressure is exerted on the water.Water transmits this pressure in all the directions.
  - The other pistons,except A, moves at the same speed which shows water has exerted pressure in all the directions.



A circular vessel fitted with 4 pistons. If piston 'A' moves inwards all other piston moves outwards.

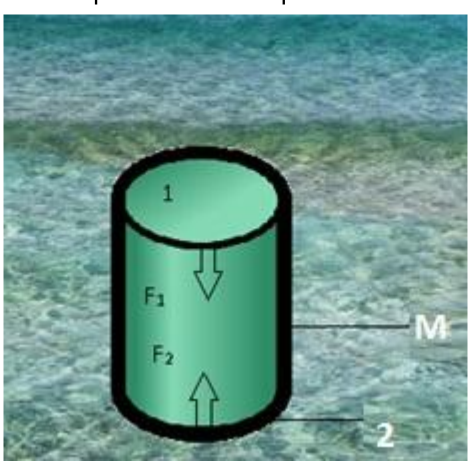
Conclusion:-

1. For a uniform fluid in equilibrium, pressure is same at all points in a horizontal plane. This means there is no net force acting on the fluid the pressure is same at all the points.
2. A fluid moves due to the differences in pressure. That means fluid will always move from a point which is at a higher pressure to the point which is at a lower pressure.

Example: - Blowing of Wind. Wind is nothing but moving air. Air is a fluid so the air moves from the region of higher pressure to the region of lower pressure.

Variation of pressure with depth

- Consider a cylindrical object inside a fluid;consider 2 different positions for this object.
- Fluid is at rest therefore the force along the horizontal direction is 0.
- Forces along the vertical direction:-
  - Consider two positions 1 and 2.
  - Force at position 1 is perpendicular to cross sectional area A, $F_1 = P_1$
  - Similarly $F_2 = P_2$
- Total force $F_{net} = -F_1 + F_2$ as F_1 is along negative y axis therefore it is –ive. And F_2 is along +ive y -axis.
- $F_{net} = (P_2 - P_1)A$
- This net force will be balanced by the weight of the cylinder(m).
- Therefore under equilibrium condition
  - F_{net}=mg=weight of the cylinder = weight of the fluid displaced.
  - =ρ Vg where ρ=density=volume of the fluid
  - =ρhAg where V=hA(h=height and A= area)
- Therefore $(P_2 - P_1)$ A=ρhAg
- $\mathbf{P_2 - P_1 = \rho h g}$,Therefore the difference in the pressure is dependent on height of the cylinder.
- Consider the top of the cylinder exposed to air therefore $P_1 = P_a$(where $P_a = P_1$ is equal to atmospheric pressure.)
- Then $\mathbf{P_2 = P_a + \rho h g}$
- Conclusion: The pressure P, at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount ρhg**.**
- The pressure is independent of the cross sectional or base area or the shape of the container.



Cylinder is inside the fluid.

Problem:- What is the pressure on aswimmer 10 m below the surface of a lake?

<u>Answer</u>:-

Here, h = 10 m and ρ = 1000 kg m^{-3}. Take g = 10 m s^{-2}

$P = P_a + \rho gh$

$= 1.01 \times 10^5$ Pa + 1000 kg $m^{-3} \times 10$ m $s^{-2} \times 10$ m

$= 2.01 \times 10^5$ Pa

≈ 2 atm

This is a 100% increase in pressure fromsurface level. At a depth of 1 km the increase in

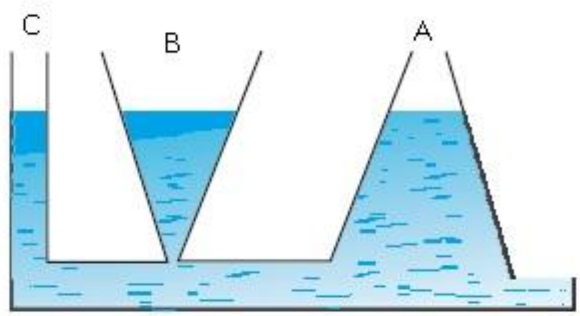
Pressure is 100 atm! Submarines are designedto withstand such enormous pressures.

Problem:- A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.
Answer:- Yes
The maximum allowable stress for the structure, P = 10^9 Pa
Depth of the ocean, d = 3 km = 3×10^3 m
Density of water, ρ = 10^3 kg/m^3
Acceleration due to gravity, g = 9.8 m/s^2
The pressure exerted because of the sea water at depth, d = ρdg

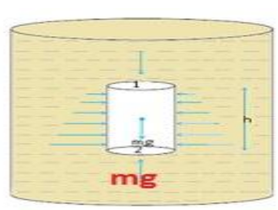
= $3 \times 10^3 \times 10^3 \times 9.8 = 2.94 \times 10^7$ Pa
The maximum allowable stress for the structure (10^9 Pa) is greater than the pressure of the sea water (2.94×10^7 Pa). The pressure exerted by the ocean is less than the pressure that the structure can withstand. Hence, the structure is suitable for putting up on top of an oil well in the ocean.

Hydrostatic Paradox

- Hydrostatic Paradox means: - hydro = water, static =at rest
  Paradox meansthat something taking place surprisingly.
- Consider 3 vessels of very different shapes (like thin rectangular shape, triangular and some filter shape) and we have a source from which water enters into these 3 vessels.
- Water enters through the horizontal base which is the base of these 3 vessels we observe that the level of water in all the 3 vessels is same irrespective of their different shapes.
- This is because pressure at some point at the base of these 3 vessels is same.
- The water will rise in all these 3 vessels till the pressure at the top is same as the pressure at the bottom.
- As pressure is dependent only on height therefore in all the 3 vessels the height reached by the water is same irrespective of difference in their shapes.
- This experiment is known as Hydrostatic Paradox.



Thethree vessels A, B and C contain differentamounts of liquids, all up to the same height



Fluid is under gravity. The effect of gravityis illustrated through pressure on a vertical cylindrical column

Atmospheric Pressure

- Pressure exerted by the weight of the atmosphere.

- Atmosphere is a mixture of different gases. All these gas molecules together constitute some weight. By virtue of this weight there is some pressure exerted by the atmosphere on all the objects.
- This pressure is known as atmospheric pressure.
- Value of atmospheric pressure at sea level is $1.01*10^5$
- $1atm = 1.01*10^5 Pa$

Problem:- What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature? Surface tension of mercury at that temperature (20°C) is 4.65×10^{-1} N m^{-1}. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.

Answer:-

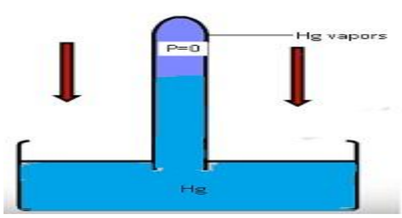
Radius of the mercury drop, $r = 3.00$ mm $= 3 \times 10^{-3}$ m
Surface tension of mercury, $S = 4.65 \times 10^{-1}$ N m^{-1}
Atmospheric pressure, $P_0 = 1.01 \times 10^5$ Pa
Total pressure inside the mercury drop

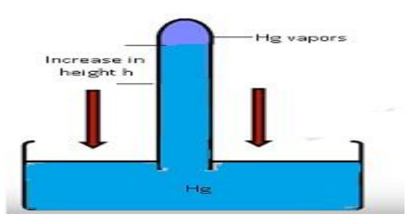
= Excess pressure inside mercury + Atmospheric pressure

$=2S/r + P_0$
$= (2x4.65x10^{-1})/(3x10^{-3})$
$= 1.0131 \times 10^5 = 1.01 \times 10^5$ Pa
Excess pressure $= 2S/r = (2x4.65x10^{-1})/3x10^{-3}$
= 310 Pa

How to measure atmospheric pressure?

- Atmospheric pressure is measured by **Mercury Barometer**.
- Mercury barometer consists of trough filled with mercury(Hg).There is a tube which also contains mercury and it is invertedinside the trough.
- The one end of tube is closed and other end of the tube is placed inverted inside the trough.
- The inverted tube which also contains mercury up to a certain level and the space above mercury in the tubeis occupied by the vapours of mercury. The pressure can be considered as 0 at this place.
- The atmosphere will exert some atmospheric pressure on the mercury level as a result the level of mercury decreases in the trough andit increases in the tube.
- This increase in level will determine how much pressure was exerted by the atmosphere.
- The pressure exerted is directly ∝to the increase in the mercury column of the tube.
- We can say that pressure at point A is same as pressure at point B.
- $P_{atm}=h\rho g$.
- It is measured in terms of how many mm of Hg rose in the column.
- Greater the height greater is the atmospheric pressure.
- When the height in this column becomes 76cm Hg we can say that the pressure applied is equal to 1atm.





After applying the pressure level of mercury rises in the tube.

Mercury Barometer

Units of Pressure:-

1.SI unit: Pascal (Pa)

- Pressure is always measured by taking sea level as the reference level. At sea level **$P=1.01*10^5$ Pa**.

2. Atmosphere (atm)
   - Reference level is at sea level.
   - Pressure equivalent of 76cm of Hg column
   - 1atm=76cm of Hg column
   - **1atm=1.01*10^5 Pa**
3. Torr
   - Pressure equivalent of 1mm of Hg column.
   - **1torr =133 Pa**
4. Bar
   - **1bar = 10^5 Pa**

Problem:- A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of 1.5×10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?

Answer

The weight that the soap film supports, W = 1.5×10^{-2} N
Length of the slider, l = 30 cm = 0.3 m

A soap film has two free surfaces.

Total length = 2l = 2 × 0.3 = 0.6 m

Surface tension, S = (Force or Weight)/2l

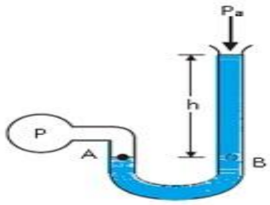
=$(1.5\text{x}10^{-2})/0.6 = 2.5\text{x}10^{-2}$N/m
Therefore, the surface tension of the film is 2.5×10^{-2} N m^{-1}.

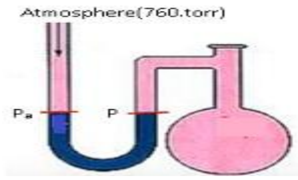
Gauge Pressure

- Pressure difference between the system and the atmosphere.
- From relation $P=P_a+\rho gh$ where P= pressure at any point, P_a = atmospheric pressure.
- We can say that Pressure at any point is always greater than the atmospheric pressure by the amount ρgh.
- $P-P_a=\rho gh$ where
- P =pressure of the system, P_a=atmospheric pressure,
- $(P-P_a)$ = pressure difference between the system and atmosphere.
- hρg = Gauge pressure.

How to measure Gauge pressure

- Gauge pressure is measured by **Open Tube Manometer**.
- Open Tube Manometer is a U-shaped tube which is partially filled with mercury(Hg).
- One end is open and other end is connected to some device where pressure is to be determined.This means it is like a system.
- The height to which the mercury column will rise depends on the atmospheric pressure. Similarly depending on the pressure of the system the height of mercury in another tube rises.
- The pressure difference between these two heights is the difference between the atmospheric pressure and system.
- This difference in pressure is the gauge pressure.
- Consider if the level of mercury column is same in both the U-tubes.
- P_{atm}=P, therefore the difference between the atmospheric pressure and the pressure of the system is 0.
- Gauge Pressure is 0.
- P_{atm} = 760torr.





Open tube manometer

Closed end manometer

Problem:- What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20 °C) is 2.50×10^{-2} N m^{-1}? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is 1.01×10^5 Pa).

Answer:

Excess pressure inside the soap bubble is 20 Pa;

Pressure inside the air bubble is

Soap bubble is of radius, r = 5.00 mm = 5×10^{-3} m
Surface tension of the soap solution, S = 2.50×10^{-2} Nm^{-1}
Relative density of the soap solution = 1.20

Density of the soap solution, $\rho = 1.2 \times 10^3$ kg/m^3
Air bubble formed at a depth, h = 40 cm = 0.4 m

Radius of the air bubble, r = 5 mm = 5×10^{-3} m
1 atmospheric pressure = 1.01×10^5 Pa
Acceleration due to gravity, g = 9.8 m/s^2
Hence, the excess pressure inside the soap bubble is given by the relation:

$P=4S/r$

$=(4x2.5x10^{-2})/5x10^{-3}$
=20Pa

Therefore, the excess pressure inside the soap bubble is 20 Pa.

The excess pressure inside the air bubble is given by the relation:

$P'=2S/r$

$=(2x2.5x10^{-2})/5x10^{-3}$
=10Pa

Therefore, the excess pressure inside the air bubble is 10 Pa.

At a depth of 0.4 m, the total pressure inside the air bubble

= Atmospheric pressure + hρg + P'

$=1.01x10^5+0.4x1.2x10^3x9.8+10$
$=1.057x10^5$Pa
$=1.06x10^5$Pa
Therefore, the pressure inside the air bubble is $1.06x10^5$Pa.

Absolute Pressure

- Absolute pressure is defined as the pressure above the zero value of pressure.

- It is the actual pressure which a substance has.
- It is measured against the vacuum.
- Absolute pressure is measured relative to absolute zero pressure.
- It is sum of atmospheric pressure and gauge pressure.
- $P = P_a + h\rho g$ where P = pressure at any point, P_a = atmospheric pressure and $h\rho g$= gauge pressure.
- Therefore **$P = P_a$ + Gauge Pressure**. Where P = absolute pressure.
- It is measured with the help of barometer.

Problem: The density of theatmosphere at sea level is 1.29 kg/m^3.Assume that it does not change withaltitude. Then how high would theatmosphere extend?

Answer:

From equation: - $P=P_a+\rho gh$

$\rho gh = 1.29\ kg\ m^{-3} \times 9.8\ m\ s^2 \times h\ m = 1.01 \times 10^5\ Pa$

$\therefore h = 7989\ m \approx 8\ km$

In reality the density of air decreases withheight. So does the value of g. The atmospheric

cover extends with decreasing pressure over100 km. We should also note that the sea level

atmospheric pressure is not always 760 mm ofHg. A drop in the Hg level by 10 mm or more isa sign of an approaching storm.

Problem:- At a depth of 1000 m in anocean (a) what is the absolute pressure?(b) What is the gauge pressure? (c) Findthe force acting on the window of 20 cm × 20 cm of a submarine at this depth, the interior of which is maintainedat sea-level atmospheric pressure. (Thedensity of sea water is $1.03 \times 10^3\ kg\ m^{-3}$,$g = 10m\ s^{-2}$.)

Answer:

Here h = 1000 m and $\rho = 1.03 \times 10^3\ kg\ m^{-3}$.

(a) From Eq. $P_2 - P_1 = \rho gh$, absolute pressure

$P = P_a + \rho gh$

$= 1.01 \times 10^5\ P_a + 1.03 \times 10^3\ kg\ m^{-3} \times 10\ m\ s^{-2} \times 1000\ m$

$= 104.01 \times 10^5\ Pa$

$\approx 104\ atm$

(b) Gauge pressure is $P - P_a = \rho gh = P_g$

$P_g = 1.03 \times 10^3\ kg\ m^{-3} \times 10\ ms^2 \times 1000\ m$

$= 103 \times 10^5\ Pa$

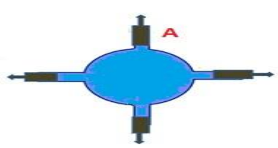
$\approx 103\ atm$

(c) The pressure outside the submarine is$P = P_a + \rho gh$ and the pressure inside it isP_a. Hence, the net pressure acting on thewindow is gauge pressure, $P_g = \rho gh$. Sincethe area of the window is $A = 0.04\ m^2$, theforce acting on it is

$F = P_g A = (10^3 \times 10^5\ Pa) \times 0.04\ m^2 = 4.12 \times 10^5\ N$

Pascal's law for transmission of fluid pressure

- Pascal's law for transmission of fluid pressure states that thepressure exerted anywhere in a confined incompressible fluid is transmitted undiminished and equally in all directions throughout the fluid.
- The above law means that if we consider a fluid which is restricted within a specific region in space and if the volume of the fluid doesn't change with the pressure,then the amount of pressure exerted will be same as the amount of pressure transmitted.
  - Consider a circular vessel which have 4 openings and along these 4 openings 4 pistons are attached.
  - When piston A is moved downwards pressure is exerted on the liquid in the downward direction, this pressure gets transmitted equally along all the directions. As a result all the other 3 pistons move equal distance outwards.

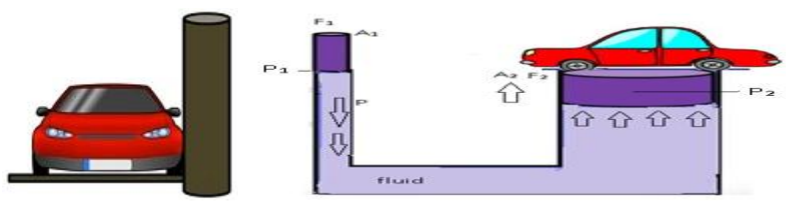


A circular vessel fitted with movable piston at all the four ends and when piston A is moved downward a pressure is exerted downward. Equal amount of pressure is exerted along all the directions as a result they will move equal distances outward.

Applications:Pascal's law for transmission of fluid pressure

Hydraulic lift:-

- Hydraulic lift is a lift which makes use ofa fluid.
  - For example: Hydraulic lifts that are used in car service stations to lift the cars.
- **Principle**: -
- Inside a hydraulic lift there are 2 platforms,one has a smaller area and the other one has a larger area.
- It is a tube like structure which is filled with uniform fluid.
- There are 2 pistons (P_1 and P_2)which are attached at both the ends of the tube.
- Cross-sectional area of piston P_1 is A_1 and of piston P_2 is A_2.
- If we apply force F_1 on P_1, pressure gets exerted and according to Pascal's law the pressure gets transmitted in all the directions and same pressure gets exerted on the other end.As a result the Piston P_2 moves upwards.
- Advantage of using hydraulic lift is that by applying small force on the small area we are able to generate a larger force.
- Mathematically:- $F_2=PA_2$
  - where F_2 = Resultant Force,A_2 = area of cross-section
  - $F_2= (F_1/A_1)A_2$ where $P=F_1/A_1$ (Pressure P is due to force F_1 on the area A_1)
  - $F_2 =(A_2/A_1)F_1$. This shows that the applied force has increased by A_2/A_1.
- Because of Pascal's law the input gets magnified.



The above figure shows the internal structure of the hydraulic lift.

Problem: A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear?

Answer:-

The maximum mass of a car that can be lifted, m = 3000 kg

Area of cross-section of the load-carrying piston, $A = 425\ cm^2 = 425 \times 10^{-4}\ m^2$

The maximum force exerted by the load,

$F = mg = 3000 \times 9.8 = 29400$ N

The maximum pressure exerted on the load-carrying piston, P = F/A

$=29400/425x10^5$

$= 6.917 \times 10^5$ Pa

Pressure is transmitted equally in all directions in a liquid. Therefore, the maximum

pressure that the smaller piston would have to bear is 6.917×10^5 Pa.

Hydraulic Brakes

- Hydraulic brakes work on the principle of Pascal's law.
- According to this law whenever pressure is applied on a fluid it travels uniformly in all the directions.
- Therefore when we apply force on a small piston, pressure gets created which is transmitted through the fluid to a larger piston. As a result of this larger force,uniformbrakingis applied on all four wheels.

- As braking force is generateddue to hydraulic pressure,theyare known as hydraulic brakes.
- Liquids are used instead of gas as liquids are incompressible.

Construction

- The fluid in the hydraulic brake is known as brake fluid.
- It consists of a master cylinder, four wheel cylinders and pipes carrying brake fluid from master cylinder to wheel cylinders.
- Master cylinder consists of a piston which is connected to pedal through connecting rod.
- The wheel cylinders consist of two pistons between which fluid is filled.
- Each wheel brake consists of a cylinder brake drum. This drum is mounted on the inner side of wheel. The drum revolves with the wheel.
- Two brake shoes whichare mounted inside the drum remain stationary.

Working

- When we press the brake pedal, piston in the master cylinder forces the brake fluid through a linkage.
- As a result pressure increases and gets transmitted to all the pipes and to all the wheel cylinders according to Pascal's law.
- Because of this pressure,both the pistons move outand transmit the braking force on all the wheels.

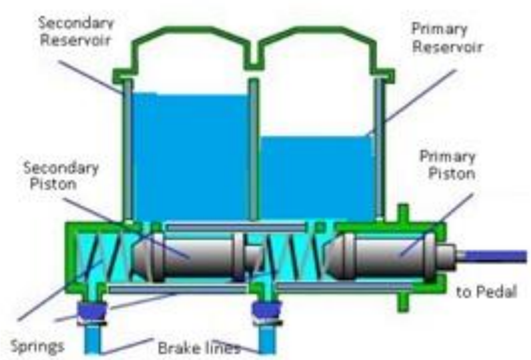
Advantages:-

- Equal braking effort to all the four wheels.
- Less rate of wear due to absence of joints.
- By just changing the size of one piston and cylinder, force can be increased or decreased.

Disadvantages:-

- Leakage of brake fluid spoils the brake shoes.
- Even the slightest presence of air pockets can spoil the whole system.





Inside of the cylinder

Problem:- Two syringes of differentcross sections (without needles) filled withwater are connected with a tightly fittedrubber tube filled with water. Diametersof the smaller piston and larger piston are1.0 cm and 3.0 cm respectively. (a) Findthe force exerted on the larger piston whena force of 10 N is applied to the smallerpiston. (b) If the smaller piston is pushed in through 6.0 cm, how much does thelarger piston move out?

Answer:

- Since pressure is transmitted undiminished throughout the fluid,

$F_2 = (A_2/A_1)\ F_1 = (3/2\ 10^{-2} m^2 / 1/2\ 10 m^{-2}\ m^2)\ 10N$
=90N.

(b) Water is considered to be perfectlyincompressible. Volume covered by themovement of smaller piston inwards is equal tovolume moved outwards due to the larger piston.

$L_1 A_1 = L_2 A_2$
$= 0.67 \times 10^{-2}$ m = 0.67 cm
Note, atmospheric pressure is common to bothpistons and has been ignored.

Problem:- In a car lift compressed airexerts a force F_1 on a small piston havinga radius of 5.0 cm. This pressure istransmitted to a second piston of radius 15 cm. If the mass of the car tobe lifted is 1350 kg, calculate F_1. What isthe pressure necessary to accomplish thistask? (g = 9.8 ms^{-2}).

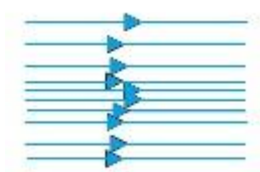
Answer:-

Since pressure is transmittedundiminished throughout the fluid,

$F_1=A_1/A_2\ F_2$
$= (5x10^{-2}\ m^2/15x10^{-2}m^2)$ 1350N x 9.8ms^{-2}
$=1470N = 1.5x10^3N$
The air pressure that will produce thisforce is

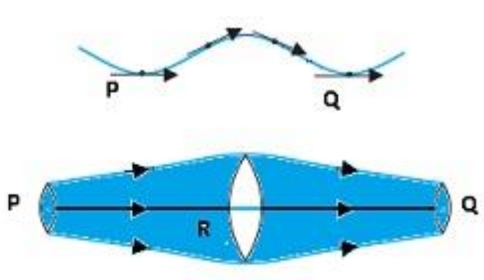
$P=F_1/A_1 = (1.5x10^3N/5x10^{-2}m^2)1.9x10^5$ Pa
This is almost double the atmosphericpressure.

Types of Fluid flow: Steady Flow



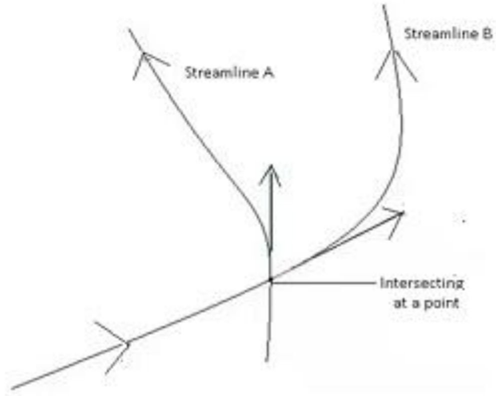
Some streamlines for fluid flow

- The flow of a fluid is said to be steady, if at any point,the velocity of each passing fluid particle remains constant within that interval of time.
- Streamline is the path followed by the fluid particle.
- It means that at any particular instant the velocities of all the particles at any point are same.But the velocity of all the particles won't be same across all the points in the space.
- Steady flow is termed as 'Streamline flow' and 'Laminar flow'.
- Consider a case when all the particles of fluid passing point A have the same velocity. This means that the first particle will have velocity V_1 and second will have velocity V_1 and so on. All the particles will have the same velocity V_1at point A.
- At point B,all particleswill have velocity V_2.
- Similarly at point C the velocity of all the particles is V_3.
- We can see that the velocity is changing from point to point but at one particular point it is same.
- No two streamlines can intersect.
- If two streamlines intersect each other, the particleswon't know which path to follow and what velocity to attain.That is why no two streamlines intersect.



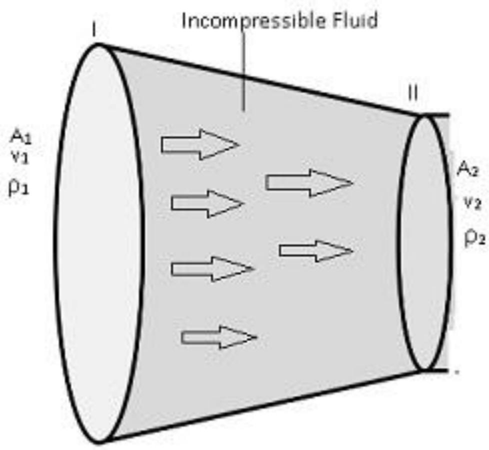
The meaning of streamlines:- (a) A typicaltrajectory of a fluid particle.

(b) A region of streamline flow.



Equation of Continuity

- According to the equation of continuity Av = constant. Where A =cross-sectional area and v=velocity with which the fluid flows.
- It means that if any liquid is flowing in streamline flow in a pipe of non-uniform cross-section area, then rate of flow of liquid across any cross-section remains constant.
- Consider a fluid flowing through a tube of varying thickness.
- Let the cross-sectional area at one end (I) = A_1 and cross-sectional area of other end (II)= A_2.
- The velocity and density of the fluid at one end (I)=v_1,$\rho_1$respectively, velocity and densityof fluid at other end (II)= v_2,ρ_2
- Volume covered by the fluid in a small interval of time Δt,across left cross-sectional is Area (I) =$A_1 x v_1 x \Delta t$
- Volume covered by the fluid in a small interval of time Δtacrossright cross-sectional Area(II) = $A_2 x\ v_2 x \Delta t$
- Fluid inside is incompressible(volume of fluid does not change by applying pressure) that is density remains sameρ_1=ρ_2. (equation 1)
- Along(I) mass=ρ_1 A_1 $v_1 \Delta t$ and along second point (II) mass = $\rho_2 A_2$ $v_2 \Delta t$
- By using equation (1). We can conclude that $\mathbf{A_1\ v_1 = A_2\ v_2}$.This is the equation of continuity.
- From Equation of continuity we can say that **Av=constant**.
- This equation is also termed as "**Conservation of mass of incompressible fluids**".



Conclusion:

1. Volume flux/Flow rate remains constant throughout the pipe. This means rate of flow of fluid of liquidis more if cross-sectional area is more, then the velocity will be less,andvice-versa.
1. But the Av will remain constant.
2. So the volume which is covered by the fluid at any cross-sectional area is constant throughout the pipe even if pipe has different cross-sectional areas.
2. The fluid is accelerated while passing from the wider cross sectional area towards the narrower area. This means if area is more the velocity is less and vice-versa.

Problem: - The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes?

Answer:-

Area of cross-section of the spray pump, $A_1 = 8\ cm^2 = 8 \times 10^{-4}\ m^2$
Number of holes, n = 40

Diameter of each hole, $d = 1\ mm = 1 \times 10^{-3}\ m$
Radius of each hole, $r = d/2 = 0.5 \times 10^{-3}\ m$
Area of cross-section of each hole, $a = \pi r^2 = \pi (0.5 \times 10^{-3})^2\ m^2$
Total area of 40 holes, $A_2 = n \times a = 40 \times \pi (0.5 \times 10^{-3})^2\ m^2$
$= 31.41 \times 10^{-6}\ m^2$
Speed of flow of liquid inside the tube, $V_1 = 1.5\ m/min = 0.025\ m/s$
Speed of ejection of liquid through the holes = V_2
According to the law of continuity, we have:

$A_1V_1=A_2V_2$
$V_2= A_1V_1/ A_2$
$= (8 \times 10^{-4}x0.025)31.61x10^{-6}$
= 0.633 m/s

Therefore, the speed of ejection of the liquid through the holes is 0.633 m/s.

Turbulent Flow:

- A fluid flow is said to be turbulent if the velocity of the particles vary at any point erratically.
- This means fluid particles are moving here and there, they are not moving in organised manner. They all will have different velocities.
- Eddies are generated by this flow.Eddies are same as ripples.
- All the particles are moving here and there randomly.

Bernoulli's Principle

- For a streamline fluid flow, the sum of the pressure (P), the kinetic energy per unit volume ($\rho v^2/2$) and the potential energy per unit volume (ρgh) remain constant.
- Mathematically:- $\mathbf{P+ \rho v^2/2 + \rho gh = constant}$
  - where P= pressure ,
  - $E./ Volume=1/2mv^2/V = 1/2v^2(m/V) = 1/2\rho v^2$
  - $E./Volume = mgh/V = (m/V)gh = \rho gh$

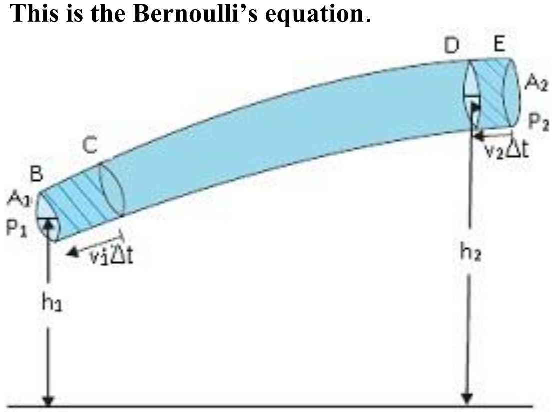
<u>Derive: Bernoulli's equation</u>

Assumptions:

1. Fluid flow through a pipe of varying width.
2. Pipe is located at changing heights.
3. Fluid is incompressible.
4. Flow is laminar.
5. No energy is lost due to friction:applicable only to non-viscous fluids.

- Mathematically: -
- Consider the fluid initially lying between B and D. In an infinitesimal timeinterval Δt, this fluid would have moved.
  - Suppose v_1= speed at B and v_2= speedat D, initial distance moved by fluid from to $C=v_1\Delta t$.
  - In the same interval Δtfluid distance moved by D to E = $v_2\Delta t$.
  - P_1= Pressureat A_1, P_2=Pressure at A_2.
  - Work done on the fluid atleft end (BC) $W_1 = P_1A_1(v_1\Delta t)$.
  - Work done by the fluid at the other end (DE)$W_2 = P_2A_2(v_2\Delta t)$
- Net work done on the fluid is $W_1 – W_2 = (P_1A_1v_1\Delta t− P_2A_2v_2\Delta t)$

- By the Equation of continuity Av=constant.
  - $P_1A_1 v_1\Delta t$ - $P_2A_2v_2\Delta t$ where $A_1v_1\Delta t = P_1\Delta V$ and $A_2v_2\Delta t = P_2\Delta V$.
- Therefore Work done = $(P_1 - P_2)\ \Delta V$equation (a)
  - Part of this work goes in changing Kinetic energy, $\Delta K = (½)m\ (v_2^2 - v_1^2)$ and part in gravitational potential energy,$\Delta U = mg\ (h_2 - h_1)$.
- The total change in energy $\Delta E = \Delta K + \Delta U = (½)\ m\ (v_2^2 - v_1^2) + mg\ (h_2 - h_1)$. (i)
- Density of the fluid $\rho = m/V$ or $m = \rho V$
- Therefore in small interval of time Δt, small change in mass Δm
  - $\Delta m = \rho\Delta V$ (ii)
- Putting the value from equation (ii) to (i)
- $\Delta E = 1/2\ \rho\Delta V\ (v_2^2 - v_1^2) + \rho g\Delta V\ (h_2 - h_1)$ equation(b)
- By using work-energy theorem: W = ΔE
  - From (a) and (b)
  - $(P_1 - P_2)\ \Delta V = (1/2)\ \rho\Delta V\ (v_2^2 - v_1^2) + \rho g\Delta V\ (h_2 - h_1)$
  - $P_1 - P_2 = 1/2\rho v_2^2 - 1/2\rho v_1^2 + \rho g h_2 - \rho g h_1$(By cancelling ΔV from both the sides).
- After rearranging we get,$\mathbf{P_1 + (1/2)\ \rho\ v_1^2 + \rho g\ h_1 = (1/2)\ \rho\ v_2^2 + \rho g\ h_2}$
- $\mathbf{P + (1/2)\ \rho v^2 + \rho g\ h}$ **= constant.**
- **This is the Bernoulli's equation**.



The flow of an ideal fluid in a pipe ofvarying cross section. The fluid in asection of length v1Δt moves to the sectionof length v2Δt in time Δt.

Bernoulli's equation: Special Cases

1. When a fluid is at rest. This means $v_1 = v_2 = 0$.
   - From Bernoulli's equation $P_1 + (1/2)\ \rho\ v_1^2 + \rho g\ h_1 = (1/2)\ \rho\ v_2^2 + \rho g\ h_2$
   - By putting$v_1 = v_2 = 0$ in the above equation changes to
     - $P_1 - P_2 = \rho g(h_2 - h_1)$. This equation is same as when the fluids are at rest.
2. When the pipe is horizontal.$h_1 = h_2$.This means there is no Potential energy by the virtue of height.
   - Therefore from Bernoulli's equation($P_1 + (1/2)\ \rho\ v_1^2 + \rho g\ h_1 = (1/2)\ \rho\ v_2^2 + \rho g\ h_2$)
   - By simplifying,$P + (1/2)\ \rho\ v^2$ = constant.

Problem:-

Water flows through a horizontal pipeline of varying cross-section.If the pressure of waterequals 6cm of mercury at a point where the velocity of flow is 30cm/s, what is the pressure at the another point where the velocity of flow is 50m/s?

Answer:-

At R_1:- v_1 = 30cm/s =0.3m/s

$P_1 = \rho g\ h = 6\text{x}10^{-2}\text{x}13600\text{x}9.8 = 7997\text{N/m}^2$

At R_2:- v_2=50cm/s=0.5m/s

From Bernoulli's equation: - $P + (1/2)\ \rho\ v^2 + \rho g\ h$=constant

$P_1 + (1/2)\ \rho\ v_1^2 = P_2(1/2)\ \rho\ v_2^2$

$7997 + 1/2\text{x } 1000\text{x } (0.3)^2 = P_2 + 1/2\text{x } 1000\text{x } (0.5)^2$

$P_2 = 7917\text{N/m}^2$

$= \rho g\ h_2 = h_2\text{x}13600\text{x}9.8$

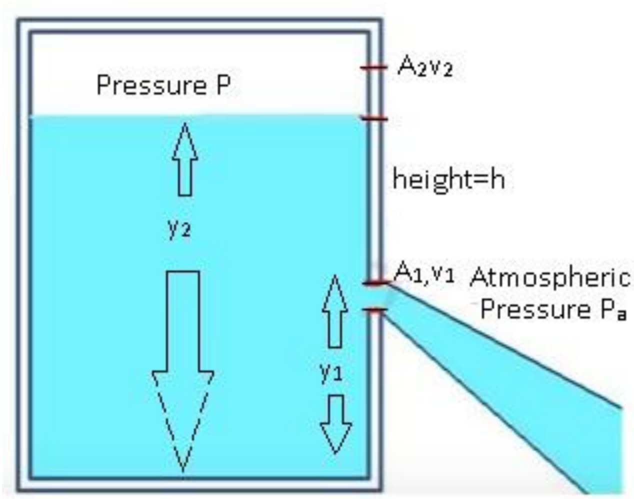
h_2 = 5.9cmHg.

Torricelli's law

- Torricelli law states that the speed of flow of fluid from an orifice is equal to the speed that it would attain if falling freely for a distance equal to the height of the free surface of the liquid above the orifice.
- Consider any vessel which has an orifice (slit)filled with some fluid.
- The fluid will start flowing through the slit and according to Torricelli law the speed with which the fluid will flow is equal to the speed with which a freely falling bodyattains such that the height from which the body falls is equal to the height of the slit from the free surface of the fluid.
- Let the distance between the free surface and the slit = h
- Velocity with which the fluid flows is equal to the velocity with which a freely falling body attains if it is falling from a height h.

<u>Derivation of the Law:-</u>

- Let A_1= area of the slit (it is very small), v_1= Velocity with which fluid is flowing out.
  - A_2=Area of the free surface of the fluid,v_2=velocity of the fluid at the free surface.
- From Equation of Continuity, Av=constant.Therefore $A_1v_1 = A_2v_2$.
  - From the figure, $A_2>>>A_1$, This implies $v_2<<v_1$(This meansfluid is at rest on the free surface), Therefore $v_2\sim 0$.
- Using Bernoulli's equation,
  - $P+ (1/2)\rho v^2+\rho gh$ = constant.
- Applying Bernoulli's equation at the slit:
  - $P_a+(1/2)\rho v_1^2+\rho gy_1$(Equation 1) where P_a=atmospheric pressure,y_1=height of the slit from the base.
- Applying Bernoulli's equation at the surface:
- $P+\rho gy_2$(Equation 2) where as $v_2=0$ therefore $(1/2)\rho v_1^2=0$, y_2=height of the free surface from the base.
- By equating(1) and (2),
- $P_a+ (1/2)\rho v_1^2+ \rho gy_1= P+\rho gy_2$
- $(1/2)\rho v_1^2 = (P-P_a) + \rho g(y_2-y_1)$
- $=(P-P_a)\rho gh$ (where $h=(y_2-y_1)$)
- $v_1^2=2/\rho [(P-P_a) + \rho gh]$
- Therefore $\mathbf{v1=\sqrt{2/\rho [(P-P_a) + \rho gh]}}$.This is the velocity by which the fluid will come out of the small slit.
- **v_1 is known as Speed of Efflux. This means the speed of the fluid outflow.**



Torricelli's law. The speed of efflux, v1,from the side of the container is given bythe application of Bernoulli's equation.

<u>Case1</u>:- The vessel is not closed it is opento atmosphere that means $P=P_a$.

- Therefore $\mathbf{v_1=\sqrt{2gh}}$.This is the speed of a freely falling body.
- This is accordance to Torricelli's law which states that the speed by which the fluid is flowing out of a small slit of a container is same as the velocity of a freely falling body.
- **<u>Case2:-</u>**Tank is not open to atmosphere but $P>>P_a$.
  - Therefore 2gh is ignored as it is very very large, hence $\mathbf{v_1= \sqrt{2P/\rho}}$.
  - The velocity with which the fluid will come out of the container is determined by the Pressure at the free surface of the fluid alone.

Problem: -Calculate the velocity of emergence of a liquid from a hole in the side of a wide cell 15cm below the liquid surface?

Answer:-

By using Torricelli'slaw $v1=\sqrt{2gh}$

$=\sqrt{2x9.8x15x10^{-2}}$ m/s

=1.7m/s

Problem:- Torricelli's barometer used mercury. Pascal duplicated it using French wine of density $984kgm^{-3}$. Determine the height of the wine column for normal atmospheric pressure.

Answer:-

Density of mercury, $\rho_1 = 13.6 \times 10^3 kg/m^3$

Height of the mercury column, $h_1 = 0.76$ m

Density of French wine, $\rho_2 = 984\ kg/m^3$

Height of the French wine column = h_2

Acceleration due to gravity, $g = 9.8\ m/s^2$

The pressure in both the columns is equal, i.e.

Pressure in the mercury column= Pressure in the French wine column

$\rho_1 h_1 g = \rho_2 h_2 g$

$h_2= \rho_1 h_1/ \rho_2$

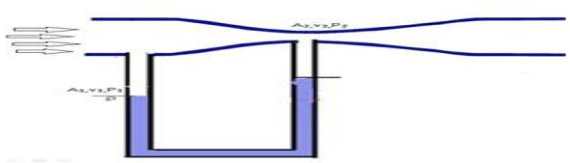
$= (13.6x10^3x0.76)/984$

= 10.5 m

Hence, the height of the French wine column for normal atmospheric pressure is 10.5 m.

Venturimeter

- Venturimeter is a device to measure the flow of incompressible liquid.
  - It consists of a tube with a broad diameter having a larger cross-sectional area but there is a small constriction in the middle.
  - It is attached to U-tube manometer. One end of the manometer is connected to the constriction and the other end is connected to the broader end of the Venturimeter.
  - The U-tube is filled with fluid whose density is ρ.
  - A_1= cross-sectional area at the broader end, v_1 = velocity of the fluid.
  - A_2=cross-sectional area at constriction, v_2= velocity of the fluid.
- By the equation of continuity, wherever the area is more velocity is less and vice-versa.As A_1 is more this implies v_1 is less and vice-versa.
- Pressure is inversely ∝ to Therefore at A_1 pressureP_1 is less as compared to pressure P_2 at A_2.
  - This implies $P_1<P_2$ as $v_1>v_2$.
- As there is difference in the pressure the fluid moves,this movement of the fluid is marked by the level of the fluid increase at one end of the U-tube.



A schematic diagram of Venturimeter

Venturimeter: determining the fluid speed

- By Equation of Continuity: -$A_1v_1=A_2v_2$.
- This implies $v_2=(A_1/A_2)v_1$ (Equation(1))
- By Bernoulli's equation:- $P_1 + (1/2)\rho v_1^2 + \rho g h = (1/2)\rho v_2^2 + \rho g h$
  - As height is same we can ignore the term ρg
  - This implies $P_1-P_2=(1/2)\rho(v_2^2- v_1^2)$
  - $=1/2\rho(A_1^2/A_2^2 v_1^2- v_1^2)$(Using equation(1)
  - $=1/2\rho v_1^2(A_1^2/A_2^2 -1)$

- $=1/2\rho v_1^2(A_1^2/A_2^2-1)$

- As there is pressure difference the level of the fluid in the U-tube changes.
- $(P_1-P_2) = h\rho_m g$where ρ_m(density of the fluid inside the manometer).
- $1/2\rho v_1^2(A_1^2/A_2^2-1)=h\rho_m g$
- $\mathbf{v_1 = 2h\rho_m g/\rho[A_1^2/A_2^2-1]^{-1/2}}$

Practical Application of Venturimeter:

1. Spray Gun or perfume bottle- They are based on the principle of Venturimeter.

- Consider a bottle filled with fluidand having a pipe which goes straight till constriction.There is a narrow end of pipewhich has a greater cross sectional area.
- The cross sectional area of constriction which is at middle is less.
- There is pressure difference when we spray as a result some air goes in ,velocity of the air changes depending on the cross sectional area.
- Also because of difference in cross sectional area there is pressure difference, the level of the fluid rises and it comes out.

Problem:- The flow of blood in a large artery of an anesthetiseddog is diverted through a Venturimeter.The wider part of the meter has a crosssectionalarea equal to that of the artery.A = 8 mm^2. The narrower part has an areaa = 4 mm^2. The pressure drop in the artery is 24 Pa. What is the speed of the blood inthe artery?

Answer: -The density of blood is 10.1 to be $1.0^6 \times 10^3$ kg m^{-3}. The ratio of the areas is(A/a) = 2.

Using Equation = $\mathbf{2h\rho_m g/\rho[A_1^2/A_2^2-1]^{-1/2}}$

$v_1=\sqrt{2x24Pa/(1060kgm^{-3}x (2^2-1))} = 0.125ms^{-1}$.

Dynamic Lift

- Dynamic lift is the normal force that acts on a body by virtue of its motion through a fluid.
- Consider an object which is moving through the fluid,and due to the motion of the object through the fluid there is a normal force which acts on the body.
- This force is known as dynamic lift.
- Dynamic lift is most popularly observed in aeroplanes.
  - Whenever an aeroplane is flying in the air, due to its motion through the fluid here fluid is air in the atmosphere.Due to its motion through this fluid, there is a normal force which acts on the body in the vertically upward direction.
  - This force is known as Dynamic lift.
- Examples:
  - Airplane wings
  - Spinning ball in air

Dynamic lift on airplane wings:-

- Consider an aeroplane whose body is streamline.Below the wings of the aeroplane there is air which exerts an upward force on the wings.As a result aeroplane experiences dynamic lift.

Magnus Effect

- Dynamic lift by virtue of spinning is known as Magnus effect.
- Magnus effect is a special name given to dynamic lift by virtue of spinning.
- Example:-Spinning of a ball.
  - Case1:-When the ball is not spinning.
    - The ball moves in the air it does not spin, the velocity of the ball above and below the ball is same.
    - As a result there is no pressure difference.($\Delta P = 0$).
    - Therefore there is no dynamic lift.

- Case2:- When the ball is moving in the air as well as spinning.
  - When the ball spins it drags the air above it therefore the velocity above the ball is more as compared to the velocity below the ball.
  - As a result there is a pressure difference; the pressure is more below the ball.
  - Because of pressure difference there is an upward force which is the dynamic lift.

Problem:- In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m s^{-1} and 63 m s^{-1} respectively. What is the lift on, the wing if its area is 2.5 m^2? Take the density of air to be 1.3 kg m^{-3}.

Answer:

Speed of wind on the upper surface of the wing, V_1 = 70 m/s

Speed of wind on the lower surface of the wing, V_2 = 63 m/s

Area of the wing, A = 2.5 m^2

Density of air, ρ = 1.3 kg m^{-3}

According to Bernoulli's theorem, we have the relation:

Where,

$P_1+1/2 (\rho V_1^2) = P_2+1/2(\rho V_2^2)$

$P_2-P_1 = 1/2 \rho (V_1^2 - V_2^2)$

P_1 = Pressure on the upper surface of the wing

P_2 = Pressure on the lower surface of the wing

The pressure difference between the upper and lower surfaces of the wing provides lift

to the aeroplane.

Dynamic Lift on the wing = (P_2-P_1) A

$=1/2 \rho (V_1^2 - V_2^2) A$

$=1.3((70)^2 – (63)^2) x2.5$

= 1512.87

= 1.51×10^3 N

Therefore, the lift on the wing of the aeroplane is 1.51×10^3 N.

Problem:- A fully loaded Boeing aircraft has a mass of $3.3x10^5$kg.Its total wing area is 500m^2.It is a level flight with a speed of 960km/h.Estimate the pressure difference between the lower and upper surfaces of the wings.

Answer:-

Weight of the aircraft= Dynamic lift

mg = (P_1-P_2) A
mg/A=ΔP

ΔP=3.3x10^5x9.8/500
=6.5x10^3N/m^2

Viscosity

- Viscosity is the property of a fluid that resists the force tending to cause the fluid to flow.
- It is analogous to friction in solids.
- Example:-
  - Consider 2 glasses one filled with water and the other filled with honey.
  - Water will flow down the glass very rapidly whereas honey won't.This is because honey is more viscous than water.
  - Therefore in order to make honey flow we need to apply greater amount of force. Because honey has the property to resist the motion.
- Viscosity comes into play when there is relative motion between the layers of the fluid.The different layers are not moving at the same pace.

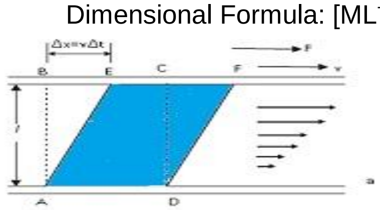
<u>Coefficient of Viscosity</u>

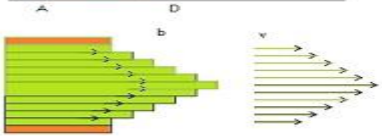
- Coefficient of viscosity is the measure of degree to which a fluid resists flow under an applied force.
- This means how much resistance does a fluid have to its motion.
- Ratio of shearing stress to the strain rate.
- It is denoted by 'η'.
- Mathematically
  - Δt=time , displacement =Δx
  - Therefore,
    - shearing stress =Δx/l where l= length
    - Strain rate=Δx/lΔt

    η=shearing stress/strain rate

    - (F/A)/(Δx/lΔt) = (Fl)/vA where Δx/t=v

    Therefore **η=(Fl)/ vA**
    I. Unit:- Poiseiulle (Pl)/Pa/Nsm^{-2}
    Dimensional Formula: [$ML^{-1}T^{-1}$]

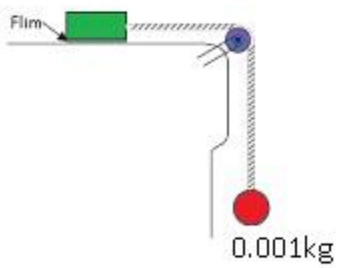




(a) A layer of liquid sandwiched betweentwo parallel glass plates in which the lower plate is fixed and the upper oneis moving to the right with velocity v

- Velocity distribution for viscous flow in a pipe.


Problem:- A metal block of area 0.10 $m^2$is connected to a 0.010 kg mass via a stringthat passes over an ideal pulley (consideredmassless and frictionless), as in Fig.A liquid with a film thickness of 0.30 mmis placed between the block and the table.When released the block moves to the rightwith a constant speed of 0.085 m s^{-1}. Findthe coefficient of viscosity of the liquid.



Answer:-
The metal block moves to the rightbecause of the tension in the string. The tension

T is equal in magnitude to the weight of thesuspended mass m. Thus the shear force F is

F = T = mg = 0.010 kg × 9.8 ms^{-2} = 9.8 × 10^{-2} N
Shear stress on the fluid = F/A =9.8 x10^{-2}/0.10
Strain rate =v/l =0.085/0.030

η= stress/strain rate

= (9.8x10^{-2}x 0.30 x10^{-3}m)/ (0.085ms^{-1}x0.10m^2)
= 3.45 × 10^{-3} Pa s

Stokes Law

- The force that retards a sphere moving through a viscous fluid is directly ∝to the velocity and the radius of the sphere, and the viscosity of the fluid.
- Mathematically:-**F =6πηrv** where
  - Let retarding force F∝v where v =velocity of the sphere
    - F ∝ r where r=radius of the sphere
    - F∝η where η=coefficient of viscosity
    - 6π=constant

  Stokes law is applicable only to laminar flow of liquids.It is not applicable to turbulent law.

  Example:-Falling raindrops
    - Consider a single rain drop, when rain drop is falling it is passing through air.
    - The air has some viscosity; there will be some force which will try to stop the motion of the rain drop.
    - Initially the rain drop accelerates but after some time it falls with constant velocity.

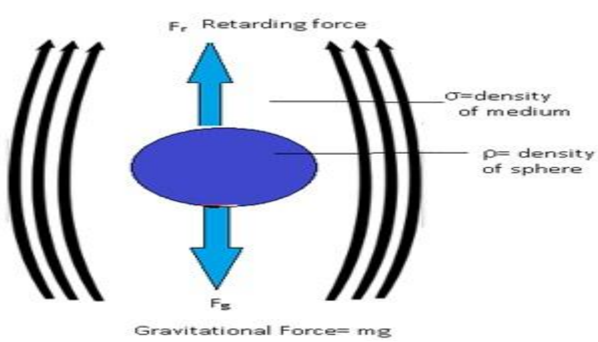
- As the velocity increases the retarding force also increases.
- There will be viscous force F_v and bind force $F_b$acting in the upward direction.There will also be $F_g$gravitational force acting downwards.
- After some time $F_g = F_r (F_v+F_b)$
- Net Force is 0. If force is 0 as a result acceleration also becomes 0.

Rain drop

Terminal Velocity

- Terminal velocity is the maximum velocity of a body moving through a viscous fluid.
- It is attained when force of resistance of the medium is equal and opposite to the force of gravity.
- As the velocity is increasing the retarding force will also increase and a stage will come when the force of gravity becomes equal to resistance force.
- After that point velocity won't increase and this velocity is known as terminal velocity.
- It is denoted by 'v_t'.Where$_t$=terminal.
- Mathematically:-
  - Terminal velocity is attained when Force of resistance = force due to gravitational attraction.
  - $6\pi\eta rv = mg$
  - $6\pi\eta rv$ = densityxVg (Because density=m/V), density=ρ – σ where ρ and σ are the densities of the sphere and the viscous medium resp.
  - $6\pi\eta rv = (\rho - \sigma) x 4/3\pi r^3 g$ where Volume of the sphere(V) $=4/3\pi r^3$
  - By simplifying
    - $=(\rho - \sigma)gx4/3r^2x1/(6\eta)$

    $v_t = 2r^2(\rho - \sigma)g/9\eta$ .This is the terminal velocity. Where($v=v_t$)



Problem: The terminal velocity of acopper ball of radius 2.0 mm fallingthrough a tank of oil at 20°C is 6.5 cm s^{-1}.Compute the viscosity of the oil at 20°C.Density of oil is 1.5×10^3 kg m^{-3}, density of copper is 8.9×10^3 kg m^{-3}.

Answer:-

Given: - $v_t = 6.5 \times 10^{-2}$ ms^{-1}, $a = 2 \times 10^{-3}$ m,$g = 9.8$ ms^{-2}, $\rho = 8.9 \times 10^3$ kg m^{-3},

$\sigma = 1.5 \times 10^3$ kg m^{-3}. From Equation: $-v_t = 2r^2 (\rho - \sigma)\, g/9\,\eta$
$= 2/9(2 \times 10^{-3}\ m^2 \times 9.8 ms^{-2} / 6.5 \times 10^{-2} ms^{-1}) \times 7.4\ 10^3\ kgm^{-3}$
$= 9.9 \times 10^{-1}\ kgm^{-1}s^{-1}$

Problem: Calculate the terminal velocity in air of an oil drop of radius 2×10^5 m from the following data $g=9.8 m/s^2$; coefficient of viscosity of air $=1.8\times10^{-5}$Pas; density of oil=900kg/m^3.The up thrust of air may be neglected?

Answer:

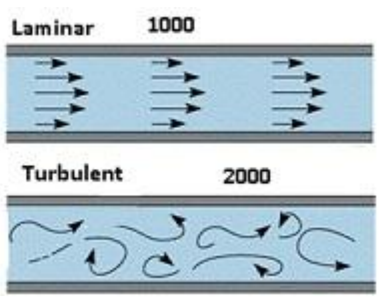
Radius $r = 2\times10^{-5}$ m
$g=9.8 m/s^2$
$\eta=1.8\times10^{-5}$Pas
$\rho = 900 kg/m^3$
$v=v_t$ when $6\pi\eta rv = mg$
$6\pi\eta rv = \rho \times 4/3 r^3 g$
Simplifying: - $v_t = 2r^2 g\rho/9\pi = (2 \times (2\times10^{-5})^2 \times 9.8 \times 900)/9 \times 1.8\times10^{-5}$
=4.36cm/s

Reynolds Number

- Reynolds number is a dimensionless number, whose value gives an idea whether the flow would be turbulent or laminar.
- Types of flow are classified as 2 types:laminar flow and turbulent flow.
- Reynolds number helps us to determine whether the flow is laminar or turbulent.
- It is denoted by R_e. where '$_e$' shows Reynolds.
- Expression: $R_e = \rho v d/\eta$;
  - where ρ = density of the fluid,
  - v=velocity of the fluid,
  - d=diameter of the pipe through which the fluid flows
  - η=viscosity of the fluid.

How does Reynolds number (R_e) distinguish laminar flow from tubular?

- If the value of Reynold's number (R_e) reaches 1000 then the flow is laminar.
- When the value of Reynold's number(R_e)is greater than 2000 then the flow is turbulent.
- If the value of (R_e)is between 1000 and 2000 then the flow is unstable.The flow is in intermediate stage.
  - At this state it has some characteristics of laminar flow and some of turbulent flow.



Alternative expression of R_e: Inertial force/force of viscosity

- By using $R_e = \rho v d/\eta$
- Multiplying both numerator and den by v:- $R_e = \rho v^2 d/\eta v$
- By rearranging, $\rho v^2/(\eta v/d)$
- Multiplying both numerator and den by A:- $R_e = \rho v^2 A/(\eta v/d)A$
  - Where
    - $\rho v^2 A$ = Inertial force
    - $(\eta v/d)A$ = Force of viscosity

(a)Calculating inertial force

- Inertial force = ma
  - $=\rho V\, xv/t = (\rho V x A x displacement)/t$
  - $=\rho v A v = \rho v^2 A$

(b)Calculating Force of viscosity:-

- Coefficient of viscosity η =stress/shearing strain
  - F/A/(x/lt)
  - F/A/v/l =Fl/Av
  - η=Fl/Av
  - F = ηAv/l
  - =(ηv/l)A (Expression is same as)

Turbulence:boon or bane

- Turbulence has both advantages and disadvantages.
- **Advantages:-**
  - Promotes mixing and increases the rates of transfer of mass,momentum and energy. For example: - Mixer and Grinder or a juice mixer.


Grinding of flour

- **Disadvantages:-**
  - Dissipates Kinetic energy in the form of heat.

Problem:- The flow rate of water froma tap of diameter 1.25 cm is 0.48 L/min.The coefficient of viscosity of water is10^{-3} Pa s. After sometime the flow rate isincreased to 3 L/min. characterise the flowfor both the flow rates.

Answer:- Let the speed of the flow be v and thediameter of the tap be d = 1.25 cm. Thevolume of the water flowing out per second is$Q = v \times \pi d^2 / 4$

$v = 4 Q / d^2$

We then estimate the Reynolds number to be

$R_e = 4 \rho Q / \pi d \eta$

$= 4 \times 103 \text{ kg m}^{-3} \times Q/(3.14 \times 1.25 \times 10^{-2} \text{ m} \times 10^{-3} \text{ Pa s})$

$= 1.019 \times 108 \text{ m}^{-3} \text{ s } Q$

Since initially

$Q = 0.48 \text{ L / min} = 8 \text{ cm}^3 / \text{s} = 8 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$,

We obtain,

$R_e = 815$

Since this is below 1000, the flow is steady.After some time when

$Q = 3 \text{ L / min} = 50 \text{ cm}^3 / \text{s} = 5 \times 10^{-5} \text{ m3 s}^{-1}$,

We obtain,

$R_e = 5095$

Liquid Surfaces

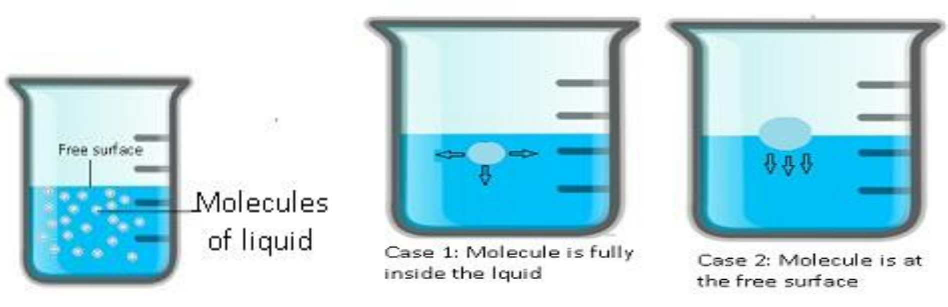
- Certain properties of free surfaces:-
- Whenever liquids are poured in any container they take the shape of that container in which they are poured and they acquire a free surface.
  - Consider a case if we pour water inside the glass it takes the shape of the glass with a free surface at the top.

- Top surface of the glass is a free surface. Water is not in contact with anything else,it is in contact with the air only.
- This is known as free surfaces.

- Liquids have free surfaces. As liquids don't have fixed shape they have only fixed volume.
- Free surfaces have additional energy as compared to inner surfaces of the liquid.

Surface Energy

- Surface energy is the excess energy exhibited by the liquid molecules on the surface compared to those inside the liquid.
- This means liquid molecules at the surface have greater energy as compared to molecules inside it.
  - Suppose there is a tumbler and when we pour water in the tumbler,it takes the shape of the tumbler.
  - It acquires free surface.
- Case 1: When molecules are inside the liquid:-
  - Suppose there is a molecule inside the water,there will be several other molecules that will attract that molecule in all the directions.
  - As a result this attraction will bind all the molecules together.
  - This results in negative potential energy of the molecule as it binds the molecule.
  - To separate this molecule huge amount of energy is required to overcome potential energy.
  - Some external energy is required to move this molecule and it should be greater than the potential energy.
  - Therefore in order to separate this molecule a huge amount of energy is required.
- Therefore a large amount of energy is required by the molecules which are inside the liquid.
- Case2: When the molecules are at the surface:-
  - When the molecule is at the surface, half of it will be inside and half of it is exposed to the atmosphere.
  - For the lower half of the molecule it will be attracted by the other molecules inside the liquid.
  - But the upper half is free. The negative potential energy is only because of lower half.
  - But the magnitude is half as compared to the potential energy of the molecule which is fully inside the liquid.
  - So the molecule has some excess energy, because of this additional energy which the molecules have at the surface different phenomenon happen like surface energy, surface tension.
- Liquids always tend to have least surface are when left to itself.
- As more surface area will require more energy as a result liquids tend to have least surface area.



<u>Surface energy for two fluids in contact</u>

- Whenever there are two fluids,in contact, surface energy depends on materials of the surfaces in contact.
- Surface energy decreases if the molecules of the two fluids attract.

- Surface energy increases if molecules of the two fluids repel.

Surface Tension

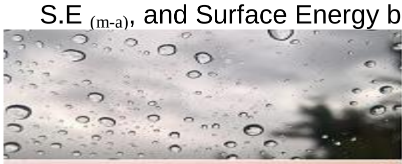
- Surface tension is the property of the liquid surface which arises due to the fact that surface molecules have extra energy.
- Surface energy is the extra energy which the molecules at the surface have.
- Surface tension is the property of the liquid surface because the molecules have extra energy.
- Surface energy is defined as surface energy per unit area of the liquid surface.
- Denoted by 'S'.
- Mathematically :-
  - Consider a case in which liquid is enclosed in a movable bar.
  - Slide the bar slightly and it moves some distance ('d').
  - There will be increase in the area, (dl) where l=length of the bar.
  - Liquids have two surfaces one on the bar and other above the bar. Therefore area=2(dl)
  - Work done for this change =Fxdisplacement.
  - Surface tension(S)=Surface Energy/area
  - Or Surface Energy=S x area
  - =Sx2dl
  - Therefore S x 2dl =F x d
- **S = F/2d**
- Surface tension is the surface energy per unit area of the liquid surface.
- It can be also defined as Force per unit length on the liquid surface.
- **Important**: -At any interface (it is a line which separates two different medium) the surface tension always acts in equal and opposite direction and it is always perpendicular to the line at the interface.

Schematic picture of molecules in a liquid, at the surface and balance of forces

(a) Moleculeinside a liquid. Forces on a molecule due to others are shown. Direction of arrows indicatesattraction of repulsion. (b) Same, for a molecule at a surface. (c) Balance of attractive (A) andrepulsive (R) forces.

Why does water stick to glass but mercury doesn't?

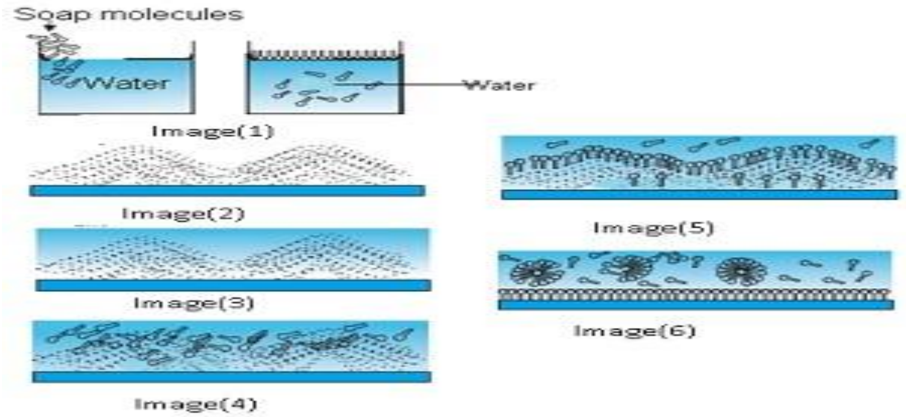
- In case of water and glass, water sticks to glass because the surface energy of waterand glass is less than the surface energy between water and air and between glass and air. $S.E_{(w-g)} < S.E_{(w-a)} + S.E_{(g-a)}$
- In case of mercury, Surface Energy between mercury and glass $S.E_{(m-g)}$, Surface energy between mercury and air $S.E_{(m-a)}$, and Surface Energy between air and glass $S.E_{(a-g)}$. $E_{(m-g)} > S.E_{(m-a)} + S.E_{(a-g)}$



How detergents work?

- Washing alone with the water can remove some of the dirt but it does not remove the grease stains. This is because water does not wet greasy dirt.

- We need detergent which mixes water with dirt to remove it from the clothes.
- Detergent molecules look like hairpin shape. When we add detergents to the water one end stick to water and the other end sticks to the dirt.
- As a result dirt is getting attracted to the detergent molecules and they get detached from the clothes and they are suspended in the water.
- Detergent molecules get attracted to water and when water is removed the dirt also gets removed from the clothes.



Detergent action in terms of whatdetergent molecules do.

In image (1) Soap molecules with head attracted to water

In image (2) greasy dirt

In image (3) water is added but dirt does not get removed

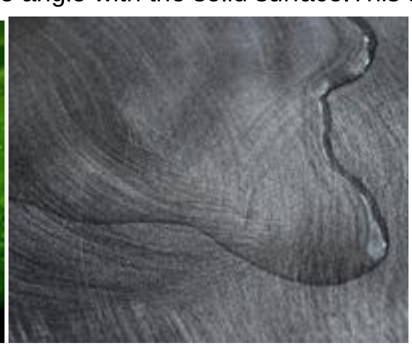
In image (4) when detergent is added, other end of the molecules get attracted to the boundary where water meets dirt.

In image (5) Dirt gets surrounded by inert end and dirt from the clothes can be removed by moving water.

In image (6) dirt is held suspended, surrounded by soap molecule,

Angle of Contact

- Angle of contact is the angle at which a liquid interface meets a solid surface.
- It is denoted by θ.
- It is different at interfaces of different pairs of liquids and solids.
- For example: - Droplet of water on louts leaf. The droplet of water(Liquid) is in contact with the solid surface which is leaf.
- This liquid surface makes some angle with the solid surface.This angle is known as angle of contact.

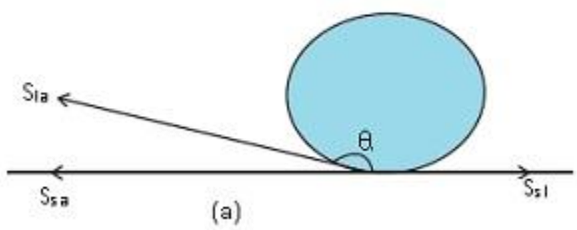



Water form a spherical shape on lotus leaf

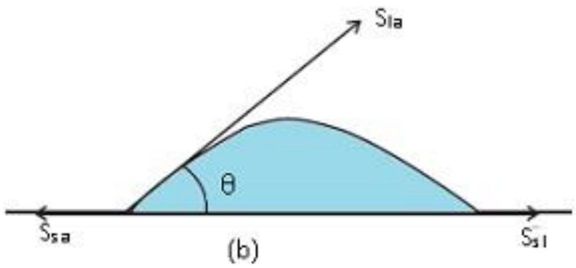
Water spilt on the table.

Significance of Angle of Contact

- Angle of contact determines whether a liquid will spread on the surface of a solid or it will form droplets on it.
  - If the Angle of contact is **obtuse**:then droplet will be formed.
  - If the Angle of contact is **acute**: then the water will spread.
- Case1: When droplet is formed
  - Consider we have a solid surface, droplet of water which is liquid and air.
  - The solid liquid interface denoted by S_{sl}, solid air interface denoted by $S_{sa}$and liquid air interface denoted by S_{la}.
  - The angle which S_{sl} makes with S_{la}. It is greater than the 90^0.
  - Therefore droplet is formed.



- Case 2: When water just spreads
  - The angle which liquid forms with solid surface is less than 90^0.



Drops and Bubbles
Why water and bubbles are drops?

- Whenever liquid is left to itself it tends to acquire the least possible surface area so that it has least surface energy so it has most stability.
- Therefore for more stability they acquire the shape of sphere, as sphere has least possible area.

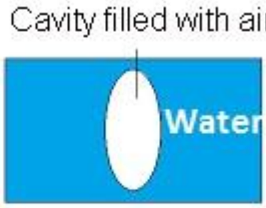

Spherical Shape

Distinction between Drop, Cavity and Bubble

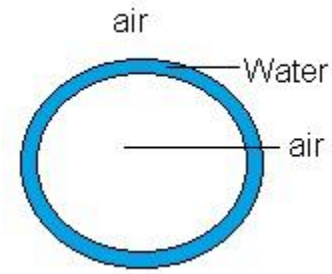
1. Drop: - Drop is a spherical structure filled with water.
1. There is only one interface in the drop.
2. The interface separates water and air.
3. Example:Water droplet.

Water droplets

2. Cavity: -Cavity is a spherical shape filled with air.
3. In the surroundings there is water and in middle there is cavity filled with air.
4. There is only one interface which separates air and water.
5. Example: - bubble inside the aquarium.



6.
7. Bubble: - In a bubble there are two interfaces.One is air water and another is water and air.
8. Inside a bubble there is air and there is air outside.
9. But it consists of thin film of water.



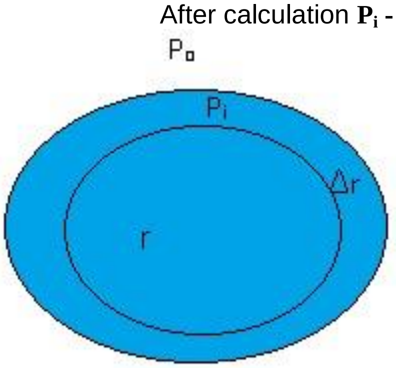
Soap bubbles

Pressure inside a drop and a cavity

- Pressure inside a drop is greater than the pressure outside.
- Suppose there is a spherical drop of water of radius 'r' which is in equilibrium.
- Consider there is increase in radius which is Δr.
- Therefore Extra Surface energy = Surface Tension(S) x area
  - $= S_{la} \times 4\pi(r+\Delta r)^2 - S_{la} \times 4\pi r^2$
  - After calculating
- **Extra Surface energy=** $8\pi r\, \Delta r\, S_{la}$
- At Equilibrium, Extra Surface energy = Energy gain due to the pressure difference
- $8\pi r\, \Delta r\, S_{la} = (P_i - P_o)\, 4\pi r^2 \times \Delta r$

where P_i= Pressure inside the drop and P_o = Pressure outside the drop.

After calculation $\mathbf{P_i - P_o = 2\, S_{la}/r}$



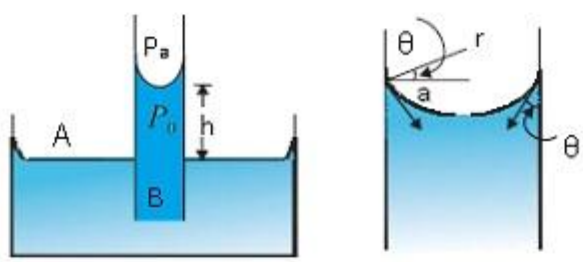
Pressure inside a Bubble

- Pressure inside a bubble is greater than the pressure outside.
- As bubble has 2 interfaces, P_i-P_o=$2S_{la}$/r x 2
- Therefore, $\mathbf{P_i - P_o = 4S_{la}/r}$

- Conclusion: - In general, for a liquid-gas interface, the convex side has a higher pressure than the concave side.

Capillary Rise

- In Latin the word capilla means hair.
- Due to the pressure difference across a curved liquid-air interface the water rises up in a narrow tube in spite of gravity.
- Consider a vertical capillary tube of circular cross section (radius a) inserted into an open vessel of water.
- The contact angle between water and glass is acute. Thus the surface of water in the capillary is concave. As a result there is a pressure difference between the two sides of the top surface. This is given by
  - $(P_i - P_o)$ =(2S/r) = 2S/(a sec θ)= (2S/a) cos θ (i)
- Thus the pressure of the water inside thetube, just at the meniscus (air-water interface)is less than the atmospheric pressure.
- Considerthe two points A and B. Theymust be at the same pressure,
  - P_0 + h ρ g = P_i = P_A (ii)
  - where ρ is the density of water,and h is called the capillary
  - h ρ g = $(P_i - P_0)$ = (2S cos θ)/a (By using equations (i) and (ii))
- Therefore the capillary rise is due to surface tension. It is larger, for a smaller radius.



(a) (b)

Capillary rise, (a) Schematic picture of anarrow tube immersed water. (b) Enlarged picture near interface.

Problem:- The lower end of acapillary tube of diameter 2.00 mm isdipped 8.00 cm below the surface of waterin a beaker. What is the pressure requiredin the tube in order to blow a hemisphericalbubble at its end in water? The surfacetension of water at temperature of theexperiments is 7.30 $\times 10^{-2}$ Nm^{-1}.1 atmospheric pressure = 1.01 × 10^5 Pa,density of water = 1000 kg/m^3, g = 9.80 m s^{-2}.Also calculate the excess pressure.

Answer:-

The excess pressure in a bubble of gasin a liquid is given by 2S/r, where S is thesurface tension of the liquid-gas interface.As there is only one liquid surface, therefore using the formula pressure is 4S/r. The radius of thebubble is r. Now the pressure outside the bubbleP_o equals atmospheric pressure plus thepressure due to 8.00 cm of water column. That is

P_o = (1.01 × 105 Pa + 0.08 m × 1000 kg m^{-3}× 9.80 m s^{-2})
= 1.01784 × 10^5 Pa
Therefore, the pressure inside the bubble is

P_i = P_o + 2S/r
= 1.01784 × 10^5 Pa + (2 × 7.3 × 10^{-2} Pa m/10^{-3} m)

$= (1.01784 + 0.00146) \times 10^5$ Pa
$= 1.02 \times 10^5$ Pawhere the radius of the bubble is taken to beequal to the radius of the capillary tube, since the bubble is hemispherical!

Thank You

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