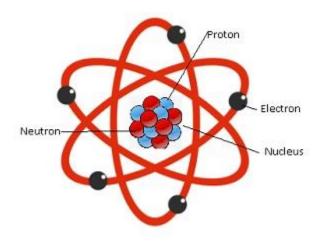
#### Introduction

- We will discuss about the nucleus which is the core of the atom.
- Nucleus is located in the centre of the atom and it is positively charged whereas electrons negatively charged and revolve around the nucleus in the orbits.
- o We will also look at the composition of the structure of a nucleus and itsdifferent properties.
- o How the present day energy crisis can be solved by using the nuclear energy.
- o The interaction between different nuclei.

### Structure of an atom



### **Composition of Nucleus**

- Nucleus consists of protons and neutrons.
- Protons are positively charged particles which are present inside the nucleus and neutrons are neutral as they don't have any charge.
- o Atomic number: -
  - Atomic number constitutes the total number of protons which are present in the nucleus of that atom.
  - It is denoted by 'Z'.
- Atomic mass:
  - o Atomic mass is the total number of neutrons and protons which are present inside the nucleus.
  - Mass of electrons is not considered while calculating the mass of the atom and only the mass of neutrons and protons are considered;
  - o Since the electrons are the lightest particles their mass is not considered.
  - It is also known as <u>Mass Number</u>.
  - It is denoted by 'A'.

### **Nucleons --> Protons + Neutrons**

- $\circ$  General representation of the element:  $-{A_Z X}$  where A = atomic mass and Z = atomic number.
  - For example:- Hydrogen 1 Where atomic number=1 and mass number =1
  - Oxygen<sup>16</sup><sub>8</sub>O where atomic number=8 and mass number =16(8 protons and 8 neutrons).

#### Measurement of Atomic mass unit

- o Mass of atom is very small as compared to the measurable masses which we see around us.
- Atomic Mass Unit (a.m.u) is used to measure mass of an atom
- o It is denoted by u.
- Atomic mass unit is defined as (1/12<sup>th</sup>) of the mass of the carbon.
- $\circ$  1 a.m.u =(1/12) x 1.992647 x10<sup>-26</sup> kg
  - $\circ$  Where 1.992647 x10<sup>-26</sup> = mass of 1 carbon atom.
- 1 a.m.u =1.67 x10<sup>-27</sup>
- o To get the exact measurement of the atomic mass, an instrument known as <u>Mass spectrometer</u> is used.

### Mass Spectrometer



<u>Problem:</u> -The three stable isotopes of neon:  $_{10}^{20}$ Ne, $_{10}^{21}$ Ne and  $_{10}^{22}$ Ne have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are

19.99 u, 20.99u and 21.99 u, respectively. Obtain the average atomic mass of neon.

#### Answer:-

Atomic mass of  $_{10}^{20}$ Ne,  $m_1$ = 19.99 u Abundance of  $_{10}^{20}$ Ne,  $\eta_1$  = 90.51% Atomic mass of  $_{10}^{21}$ Ne,  $m_2$  = 20.99 u Abundance of  $_{10}^{21}$ Ne,  $\eta_2$  = 0.27% Atomic mass of  $_{10}^{22}$ Ne,  $m_3$  = 21.99 u Abundance of  $_{10}^{22}$ Ne,  $\eta_3$  = 9.22% The average atomic mass of neon is given as:  $m = (m_1 \ \eta_1 + m_2 \ \eta_2 + m_3 \ \eta_3) / (\eta_1 + \eta_2 + \eta_3)$ = (19.99x90.51+20.99x0.27+21.99x9.22)/ (90.51+0.27+9.22) =20.1771u

#### **Nuclei types**

- 1. **Isotopes:** Two nuclei with the same atomic number and different mass number are isotopes of each other.
  - 14. For example: There are 3 isotopes of carbon(C) having same atomic number 6 but their mass numbers are differenti.e. 12,13 and  $14.\binom{12}{6}C$ ),  $\binom{13}{6}C$ ),  $\binom{14}{6}C$ ).
- 2. Isobars: -The nuclei which have different atomic number but same mass number are known as isobars.
- 14. For example: Nitrogen ( $^{14}_{7}$ N) and Carbon ( $^{14}_{6}$ C)areboth isobars as their mass numbers aresame which is 14 but their atomic numbers are 7 and 6 respectively.
- 3. Isotones: -Isotones are those nuclei which have different atomic number but same number of neutrons.
- 0. For example: -Boron ( $^{12}_{5}$ B) and Carbon ( $^{13}_{6}$ C).
- 1. Boron: Atomic number = 5 and mass number =12.
- 2. Carbon: Atomic number = 6and mass number =13.
- 3. But the number of neutrons in Boron = (12-5=7) and number of neutrons in carbon = (13-6=7) are same.

4. Nuclide: - Nuclides are collection of nuclei with same atomic number having same number of neutrons.

#### **Charge on a Nucleus**

- Nucleus is positively charged and consists of protons which are (+ively) charged and neutrons are neutral. As a whole nucleus has to be positively charged.
- o Charge on the nucleus is such that the entireatom is electrically neutral as a whole.
- o Atom constitutes of electrons, protons and neutrons.
  - Consider an element zX
    - Where Z= atomic number, (the number of protons =Z and the number of electrons =Z).
  - Total charge on all the electrons e = -(Ze).

Total charge on the nucleus has to be equal and opposite of the charge on electron i.e. it should be =+ (Ze)for the atom to be electrically neutral.

#### Size of Nucleus

- Rutherfordperformed an experiment which proved that the size of the nucleus is extremely small.
- o In Rutherford scattering experiment a beam of alpha particles were made to pass through a small thin gold foil.
- Very few alpha particles were deflected.
- The alpha particles got deflected because of repulsion with the nucleus as alpha particles are positively charged. They get repelled because they are both positively charged.
- o Very small number of alpha particles got deflected provingthat nucleus is very small in size.
- o It was found that the radius of a nucleus(R) of mass number A is given as :
  - o  $R=R_0A^{1/3}$  where A = mass number and  $R_0$ =constant.
- - $V = (4/3)\pi R^3$ , Also  $R \propto (A)^{1/3}$
  - o => (R)<sup>3</sup>∝A
  - Therefore  $V \propto (R)^3 \propto$
- o Density of nucleus is independent of mass number.

Problem:- Given the mass of iron nucleus as 55.85u and A=56, find the nuclear density?

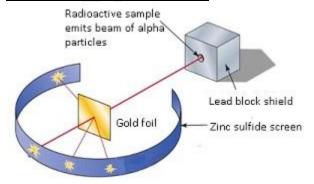
**Answer:-** Given: -  $m_{Fe} = 55.85$ ,  $u = 9.27 \times 10^{-26}$  kg

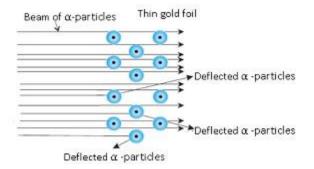
Nuclear density =mass/volume

- =  $(9.27 \times 10^{-26})/((4 \pi /3)(1.2 \times 10^{-15})^3) \times (1/56)$
- $= 2.29 \times 10^{17} \text{ kg m}^{-3}$

The density of matter in neutron stars (an astrophysical object) iscomparable to this density. This shows that matter in these objectshas been compressed to such an extent that they resemble a bignucleus.

#### Rutherford's Gold Foil Experiment





### Einstein's Mass-Energy equivalence

- o According to Einstein mass is also a form of energy.
- Mass energy can also be converted into other forms of energy.
- Einstein gave mass-energy equivalence relation as: E=mc².
- Any object which has got mass 'm' has mass energy associated with it and it is given as mc<sup>2</sup>.
- o This relation helps in understanding nuclear masses and interaction of nuclei with each other.

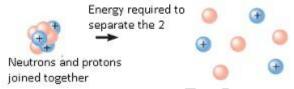
**<u>Problem:-</u>** Calculate the energy equivalent of 1 g of substance.

<u>Answer:-</u> Energy,  $E = 10^{-3} \times (3 \times 10^{8})^{2} J$  $E = 10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13} J$ 

Thus, if one gram of matter is converted to energy, there is a release of enormous amount of energy.

#### **Nuclear binding energy**

- o Nuclear binding energy is the energy required to hold an atom's protons and neutrons together in the nucleus.
- Energy required holding neutrons and protons together therefore keeps the nucleus intact.
- o It can also be defined as the energyneeded to separate the nucleons from each other.



- Importance of nuclear binding energy describes how strongly nucleons are bound to each other. By determining its value we will come to know whether the neutrons and protons are tightly or loosely bound to each other.
- If nuclear binding energy is high -> high amount of energy is needed to separate the nucleons this means nucleus is very stable.
- o If nuclear binding energy is low -> low amount of energy is needed to separate the nucleons this means nucleus is not very stable.

### Mass defect:-

- Mass defect is the difference in the mass of nucleus and its constituents (neutrons and protons).
- $\circ$  It is denoted by  $\Delta M$ .
- O Mathematically :-  $\Delta M = [Z m_p + (A-Z) m_n] M$ 
  - $_{\odot}$  Where m<sub>p</sub>=mass of 1 proton, Z=number of protons,(A-Z)= mass of neutrons, m<sub>N</sub> = mass of 1 neutronand M =nuclearmass of the atom.
- For example: -(<sup>16</sup><sub>8</sub>O)àOxygen atom has 8 protons and 8 neutrons.

- o Mass of 8 protons à (8x1.00866) u and Mass of 8 neutrons à(8x1.00727) u.
- o Therefore Oxygen nucleus à(8p+8n) à8(1.00866 + 1.00727) = 16.12744u.
- $\circ$  From spectroscopy ->Atomic mass of ( $^{16}_{8}$ O) =15.9949u.
- o Mass of 8 electrons = (8x0.00055) u.
- o Therefore Nuclear mass of  $\binom{16}{8}$ O) = (15.9949 (8x0.00055)) = 15.99053u.
- Nuclear mass is less than sum of the masses of its constituents.
- This difference in mass is known as mass defect.
- It is also known as excess mass.
- Relation between Mass defect and Nuclear binding energy:-
  - Nuclear binding energy is denoted by E<sub>b</sub>.
  - $\circ$  E<sub>b</sub>=  $\Delta$ Mc<sup>2</sup>

 $= 127.5 \text{ MeV/c}^2$ 

- O Where  $E_h$  = nuclear binding energy, ΔM=mass defect.
- As there is difference in the mass so there is energy associated with it. This energy is known as nuclear binding energy.
- Nuclear binding energy is a measure of how well a nucleus is held together.

<u>Problem:-</u> Find the energy equivalent of one atomic mass unit, first in Joules and then in MeV. Using this, express the mass defect of ( $_8^{16}$ O) in MeV/ $_c^2$ .

```
Answer:- _1u = _1.6605 \times 10^{-27} kg To convert it into energy units, we multiply it by _2 and find that energy equivalent = _1.6605 \times 10^{-27} \times (2.9979 \times 10^8)^2 kg m²/s² = _1.4924 \times 10^{-10} J = _1.4924 \times 10^{-10} J (_1.602 \times 10^{-19}) eV = _1.4924 \times 10^{-10} (_1.602 \times 10^{-19}) eV = _1.4924 \times 10^{-10} eV
```

The energy needed to separate  $\binom{8}{8}$  into its constituents is thus 127.5 MeV/c<sup>2</sup>.

<u>Problem:-</u> Find the energy equivalent of one atomic mass unit, first in Joules and then in MeV. Using this, express the mass defect of  $^{16}$  <sub>8</sub>O in MeV/c<sup>2</sup>.

```
Answer:- 1u = 1.6605 \times 10^{-27} \text{ kg}

To convert it into energy units, we multiply it by c^2 and find that energy equivalent = 1.6605 \times 10^{-27} \times (2.9979 \times 10^8)^2 \text{ kg m}^2/\text{s}^2

= 1.4924 \times 10^{-10} \text{ J}

= (1.4924 \times 10^{-10} \text{ J})/(1.602 \times 10^{-19}) \text{ eV}

= 0.9315 \times 10^9 \text{ eV}

= 931.5 \text{ MeV}

Or, 1u = 931.5 \text{ MeV/c}^2

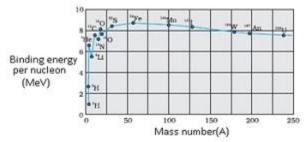
For ^{16}_{8}\text{O}, \Delta M = 0.13691 \text{ u} = 0.13691 \times 931.5 \text{ MeV/c}^2

= 127.5 \text{ MeV/c}^2
```

The energy needed to separate  ${}^{16}_{8}O$  into its constituents is thus 127.5 MeV/ $c^2$ .

#### Nuclear binding energy per nucleon

- Nuclear binding energy per nucleon is defined as the average energy per nucleon needed to separate a nucleus into its individual constituents.
- It is denoted by E<sub>hn</sub>.
- o Experimentally there was a graph plotted between binding energy per nucleon and the mass number(A).



The binding energy per nucleon as a function of mass number.

- Following are the observations from the graph:-
- 1. Initially the graph was increasing. This implies that  $E_{bn}$  is very less for lesser mass number.
- 2. In the middle range the  $E_{bn}$  becomes constant. This means  $E_{bn}$  is independent of mass number.
- 3. In the end  $E_{bn}$ starts decreasing. This shows  $E_{bn}$  is less when mass number is more.

<u>Problem</u>: - Obtain the binding energy (in MeV) of a nitrogen nucleus ( $^{14}_{7}$ N), given m ( $^{14}_{7}$ N) =14.00307u? <u>Answer:</u>-Atomic mass of ( $^{14}_{7}$ N) nitrogen, m = 14.00307 u

A nucleus of (147N) nitrogen contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus,  $\Delta m = 7m_H + 7m_n - m_H$ 

Where,

Mass of a proton,  $m_H = 1.007825 u$ 

Mass of a neutron, m<sub>n</sub>= 1.008665 u

Therefore,  $\Delta m = (7 \times 1.007825 + 7 \times 1.008665 - 14.00307)$ 

= 7.054775 + 7.06055 - 14.00307

= 0.11236 u

But 1 u =  $931.5 \text{ MeV/c}^2$ 

 $\Delta m = 0.11236 \times 931.5 \text{ MeV/c}^2$ 

Hence, the binding energy of the nucleus is given as:

 $E_b = \Delta mc^2$ 

Where, c = Speed of light

 $E_b = 0.11236 \times 931.5 (MeV/c^2)/c^2$ 

= 104.66334 MeV

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

### Deriving Nuclear force from E<sub>bn</sub>

- 1. Lighter nuclei: -
  - 1. In the initial part of the graph A(mass number) is less therefore E<sub>bn</sub> is also less. As a result lesser energy is required to separate the nucleons.
  - 2. This showsnuclei areunstable.

- 3. The nuclei are unstable and in order to become stable lighter nuclei combine with each other to form heavier nuclei.
- 4. Let the energy of heavier nuclei formed is E'<sub>bn</sub> and of lighter nuclei be E<sub>bn</sub>. This implies E'<sub>bn</sub>> E<sub>bn</sub>.
- 5. Energy is released when 2 lighter nuclei combine together to form a heavier nuclei.
- 6. This process is known as Nuclear Fusion.
- 2. For heavier nuclei:-
  - 1. Mass number is very high and  $E_{bn}$  is very less.
  - 2. In order to become stable the heavier nuclei will split into 2 lighter nuclei.
  - 3. Energy associated with heavier nuclei =E<sub>bn</sub> and energy associated with 2 lighter nuclei =E'<sub>bn</sub>.
  - 4. This implies  $E'_{bn} > E_{bn}$ . Energy is released in this process by the heavier nuclei in order to attain stability.
  - 5. This process is known as Nuclear Fission.
- 3. Constancy of E<sub>bn</sub>in the mid-range of A:-

Mass of a neutron,  $m_n = 1.008665 u$ 

- 1. In this portion the mass number is increasing due to whichnumber of nucleons also increase.
- 2. The force which is present between the nucleons is of short range. The strength of the force decreases as the distance increases.
- 3. The nucleons are getting affected by their nearest neighbouring nucleons and not by the nucleons which are far away.
- 4. As a result E<sub>bn</sub> remains constant.
- 5. But when there are too many nucleons E<sub>bn</sub> suddenly starts decreasing.

```
<u>Problem: -</u> Obtain the binding energy of the nuclei (<sup>56</sup><sub>26</sub>Fe) and (<sup>209</sup><sub>83</sub>Bi) in units of MeV from the following data: m
    \binom{56}{26}Fe) = 55.934939u and m \binom{209}{83}Bi) = 208.980388 u.
Answer:- Atomic mass of (^{56}_{26}Fe), m_{Fe} = 55.934939 u
\binom{56}{26}Fe) nucleus has 26 protons and (56 - 26) = 30 neutrons
Hence, the mass defect of the nucleus, \Delta m = ((26 \times m_H) + (30 \times m_D) - m_{Fe})
Mass of a proton, m_H = 1.007825 u
Mass of a neutron, m_n = 1.008665 u
\Delta m = ((26 \times 1.007825 + 30 \times 1.008665) - 55.934939)
= (26.20345 + 30.25995 - 55.934939)
= 0.528461 u
But 1 u = 931.5 \text{ MeV/c}^2
Therefore, \Delta m = 0.528461 \times 931.5 \text{ MeV/c}^2
The binding energy of this nucleus is given as:
E_{b1} = \Delta mc^2
Where, c =Speed of light
E_{b1} = 0.528461 \times 931.5 (MeV/c^2)/c^2
= 492.26 MeV
Average binding energy per nucleon= (492.26/56)=8.76 MeV
Atomic mass of (^{209}_{83}Bi), m_2 = 208.980388 u
\binom{209}{83}Bi) nucleus has 83 protons and (209 - 83) = 126 neutrons.
Hence, the mass defect of this nucleus is given as:
\Delta m' = (83 \times m_H + 126 \times m_n) - m_2
Where,
Mass of a proton, m_H = 1.007825 u
```

```
\Delta m' = (83 \times 1.007825 + 126 \times 1.008665 - 208.980388)
= 83.649475 + 127.091790 - 208.980388
= 1.760877 u
But 1 u = 931.5 MeV/c<sup>2</sup>
Therefore, \Delta m' = 1.760877 \times 931.5 (MeV/c<sup>2</sup>) x c<sup>2</sup>
Hence, the binding energy of this nucleus is given as:
E_{b2} = \Delta m' c^2
= 1.760877 × 931.5(MeV/c<sup>2</sup>) x c<sup>2</sup>
= 1640.26 MeV
Average binding energy per nucleon = (1640.26/209) =7.848MeV
```

#### **Nuclear force**

- o The force with which the nucleons are bound together is known as nuclear force.
- o It is the strong attractive force that binds the nucleons together.
- When the nuclear force is compared to other forces of nature like gravitational or coulomb's force etc.it is the strongest of all the forces.
- As protons are positively charged they repel each other. This force of repulsion is given by Coulomb's force of repulsion.
- o This nuclear force is stronger than the coulomb's force so it overcomes the force of repulsion.
- o This is the reason neutrons and protons are held together inside the nucleus.
- o It is independent of electric charge. Magnitude of nuclear force is same between proton-proton, proton-neutron or neutron-neutron.
- Nuclear force cannot be given mathematically.

<u>Problem</u>: - The neutron separation energy is defined as the energy required removing a neutronfrom the nucleus. Obtain the neutron separation energies of the nuclei ( $^{41}_{20}$ Ca) and ( $^{27}_{13}$ Al) from the following data: m ( $^{40}_{20}$ Ca) = 39.962591 u, m ( $^{41}_{20}$ Ca) = 40.962278 um ( $^{26}_{13}$ A) = 25.986895 u and m( $^{27}_{13}$ A) = 26.981541 u.

#### Answer:

```
For (41<sub>20</sub>Ca): Separation energy=8.363007 MeV
(27<sub>13</sub>AI): Separation energy=13.059 MeV
\binom{1}{0}n) is removed from a \binom{41}{20}Ca).
For a neutron nucleus. The corresponding nuclear reaction can be written as:
^{41}_{20}Ca --> ^{40}_{20}Ca + _{0}^{1}n
It is given that:
m (^{40}_{20}Ca) Mass = 39.962591 u,
m (41<sub>20</sub>Ca) Mass = 40.962278 u
m \binom{1}{0} n) Mass = 1.008665 u
The mass defect of this reaction is given as:
Therefore, \Delta m = m(^{40}_{20}Ca) + (_0^1n) - m(^{41}_{20}Ca)
= (39.962591+1.008665-40.962278)u =0.008978u
But 1u=931.5 (MeV/c^2)
Therefore,\Delta m = 0.008978 \times 931.5 \text{ (MeV/c}^2\text{)}
Hence, the energy required for neutron removal is calculated as:
=0.008978 x931.5 =8.363007MeV
```

For  $\binom{27}{13}$ Al), the neutron removal reaction can be written as:

 $^{27}_{13}AI --> ^{26}_{13}AI + _{0}^{1}n$ 

It is given that:

 $m(^{27}_{13}AI)Mass = 26.981541 u$ 

m (26<sub>13</sub>Al) Mass =25.986895u

The mass defect of this reaction is given as:

 $\Delta m = m \binom{26}{13} AI + m \binom{1}{0} - m \binom{27}{13} AI$ 

=25.986895 + 1.008665 -26.981541

=0.0414019u

 $=0.0414019 \times 931.5 (MeV/c^2)$ 

Hence, the energy required for neutron removal is calculated as:

E=∆mc²

=(0.0414019 x 931.5) =13.059MeV

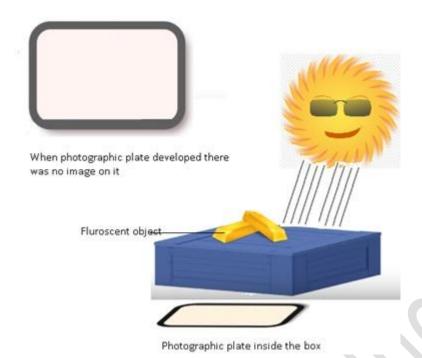
### Radioactivity

- o Radioactivity is a nuclear phenomenon in which an unstable nucleus undergoes decay to form stable nuclei.
- Radioactivity was discovered accidentally by a scientist Henry Becquerel (1896).

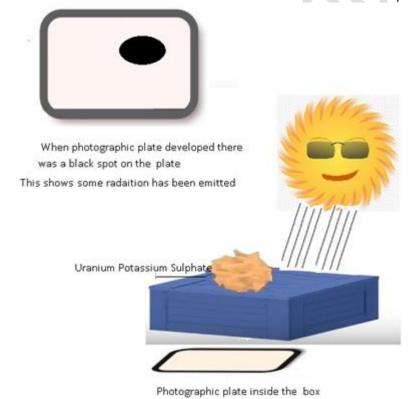
#### **Henry Becquerel**



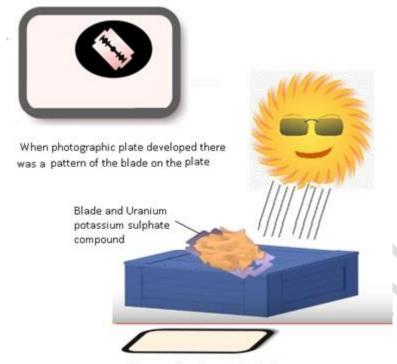
- Only those nuclei which are unstable will show the phenomenon of radioactivity.
- Fluorescence is a property shown by fluorescent objects; when visible light falls on a fluorescent object then these objects will also start emitting light.
- Henry Becquerel performed experiments on some fluorescent objects and he gave a hypothesis that fluorescent objects along with the visible light also emit some kind of radiation as well.
- o Experimental set-up:-
  - He took a photographic plate and placed inside a box.He placed a fluorescent object on the top of the box and allowed the sun light to fall on it.
  - As the box is opaque it won't allow the rays of light to pass through it.
  - When the photographic plate will be developed there should be no traces of image on the plate.
  - He repeated the experiment by taking different compounds and observed there was no image formed on the photographic film.
  - This showed that sunlight is the only ray which is emitted in the above case and as the plate was covered with opaque object so the sunlight is not able to reach the plate.



- He then took the compound of uranium known as uranium potassium sulphate.
- o When the photographic plate was developed there was a black spot on the photographic plate.
- o This shows that some radiation was emitted that reached the photographic plate.



- o In order to know from where the radiation was emitted, he kept a blade on top of a box and uranium potassium sulphate compound was placed above the blade.
- After performing the experiment, when the photographic plate was developed, the photo of blade was seen on the plate.
- This shows that the radiation was emitted by the uranium potassium sulphate compound.



Photographic plate inside the box

- o Accidentally while he was performing the experiment it was not sunny that day.
- The result which he got was same i.e. when the photographic plate was developed there was the same image of the blade on the plate.

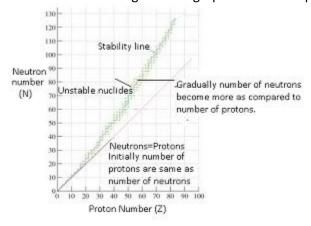
#### Conclusion:-

- The radiation was emitted by the uranium potassium sulphate compound and sunlight did not play any role in emitting the radiations.
- After series of experiments he proved that it was uranium alone which was responsible for emitting the radiations.
- He concluded that there are certain elements which emit radiation on their own.
- o This phenomenon was named as radioactivity.

### Radioactive nuclides: Nuclear stability

- Experimentally it was observed that the number of neutrons varies with the number of protons as we move higher in the periodic table.
- For the elements which have lesser atomic number (e.g.: O, C) the number of neutrons and the number of protons are equal.
- o From the graph we can see the straight line shows the number of neutrons is equal to the number of protons.
- But as the atomic number increases gradually the number of neutrons become more than the number of protons.

- o In the graph there is a specific pattern like zigzag is shown by the stable nuclides and some which don't show this pattern represents the unstable nuclides.
- The nuclides which are unstable emit some radiations to become; this phenomenon is known as radioactivity and the nuclides are known as radioactive nuclides.
- Nuclides to the left of the graph are stable which shows they have excess neutrons.
- o Nuclides to the right of the graph have excess protons. Such nuclides are known as radioactive nuclides.



#### **Radioactive Decay**

- o Radioactive decay is a phenomenon in which unstable nucleus decay to form stable nuclei.
- o There are 3 types of radioactive decay by which a unstable nuclei becomes stable nuclei:-
  - Alpha decay:
    - o In alpha decay  $\alpha$  particle (helium nucleus) is emitted.
  - Beta decay:
    - o In beta decay electrons or positrons are emitted.
    - o If electrons are emitted then it is known as β decay; and when positrons are emitted then the decay is known as β
  - Gamma decay:-
    - In gamma decay high energy photons(γ rays) are emitted.
  - Law of radioactive decay:

    - Consider a sample of radioactive nuclei (Uranium). The sample of uranium will have several
      uranium nuclei and they will undergo radioactive decay to convert into relatively stable lighter
      nuclei.
    - That means the number of uranium nuclei will gradually keep on decreasing with time.
    - For example:-
      - If in a sample there are 10 uranium nuclei and in another 100 uranium nuclei the rate of decay will be slower where there are 10 uranium nuclei as compared to 100 uranium nuclei.

### Generalized expression for law of radioactive decay:-

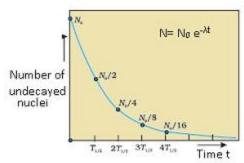
Suppose there are 'N' active nuclei at any instant of time ('t').

In small interval of time 'dt' ='dN' (number of nuclei decay).

Rate of decay  $(dN/dt) \propto N$  (number of nuclei which are present in the sample).

 $(dN/dt) = -(\lambda N)$ 

- $\circ$  Where  $\lambda$  = decay constant and (-ive) sign means the number of nuclei is gradually decreasing with time.
- $\circ$  => (dN) =- $\lambda$  dt, integrating both the sides,
- $\circ =>_{N0}^{N} \int (dN)/N = -\lambda_0^{t} \int N dt$ 
  - o Where N₀=number of nuclei at t=0 and N= number of nuclei after time t.
- $\circ$  [In N]<sub>N0</sub><sup>N</sup> = - $\lambda$ [t]<sub>0</sub><sup>t</sup>
- $\circ$  (In N<sub>0</sub> In N) =- $\lambda$ t
- $\circ = \ln(N/N_0) = -\lambda t$
- $\circ => (N/N_0) = e^{-\lambda t}$
- $\circ$  =>N = N<sub>0</sub> e<sup>- $\lambda$ t</sup> always N<sub>0</sub>> N.
- o Radioactive law is also known as <u>exponential decay law</u> as number of nuclei decreases exponentially which can be seen from the graph if plotted between N and t.
- o It is followed only by radioactive nuclei.



Exponential decay of a radioactive species. After a lapse of  $T_{1/2}$  population of the given species drops by a factor of 2.

### Terminologies related to Radioactive decay

- 1. Decay Rate: -
  - 1. Decay rate is defined as the number of nuclei decaying per unit time.
  - 2. Denoted by 'R'.
  - 3. Mathematically:- R = -(dN/dt)
    - Where dN=change in the number of nuclei with time and (-ive) sign shows the number of nuclei decreasing.
    - o From Exponential decay law  $N = N_0 e^{-\lambda t}$ , therefore
    - $\circ$  R=(d/dt)(N<sub>0</sub> e<sup>- $\lambda$ t</sup>)
- $\circ =- N_0 \left[ e^{-\lambda t} (-\lambda) \right]$
- $\circ$  R= N<sub>0</sub> $\lambda$  e<sup>- $\lambda$ t</sup>;
- o At t=0,  $R_0 = N_0 \lambda$  (equation (1))
- Therefore  $R=R_0 e^{-\lambda t}$ =>decay constant also changes exponentially.
- 2. Activity of radioactive sample:-
  - 1. Activity of radioactive sample is defined as the total decay rate R of a sample of one or more radio nuclides.
  - 2. Decay rate of whole sample is considered.
  - 3. I. unit: Becquerel(Bq); 1Bq = (1 decay/sec).
  - 4. Other unit: Curie(Ci); 1C i=3.7 x10<sup>10</sup>
- 3. Half-life of radioactive sample:-

- 1. Half life of radioactive sample is defined as the time at which number of nuclei reduces to one-half of their initial values.
- 2. It is denoted by  $t(_{1/2})$ .
- 3. Half-life tells how long radioactive nuclei can last.
- 4. Mathematical expression:-
  - From radioactive law ,  $N=N_0e^{-\lambda t}$  (equation(1))
  - Initially number of sample = N<sub>0</sub>.
  - Later number of samples will become= $(N_0/2)$ .
  - Therefore from equation(1)-  $(N_0/2) = N_0 e^{-\lambda t}$
  - $\circ =>e^{-\lambda t}(1/2)=(1/2)$
  - $\circ$  =>-  $\lambda t_{(1/2)} = ln(1/2)$  (taking log on both sides)
  - $\circ$  =>-  $\lambda t_{(1/2)} = \ln(1) \ln(2)$
  - $\circ$  => $\lambda t_{(1/2)}$  = In(2) (using In (1) =0).
  - $\circ$  => $t_{(1/2)}$  = (0.693/ $\lambda$ )
- 5. Mean-life of radioactive sample:-
  - 0. Mean life is defined as the average life of a nuclei in the radioactive sample.
  - 1. Any nuclei can decay at any interval of time.
  - 2. It is denoted by t<sub>av</sub>.
  - 3. Mathematically:-
    - Let number of radioactive sample at  $t=0 = N_0$ .
    - From the law of radioactive decay (dN/dt) =-λN
    - => dN= -λNdt (these many nuclei will decay in time dt).
    - Therefore number of nuclei which decay between t and (t+dt) =λNdt.
    - But of the nuclei will decay fast and some will slowly decay.
    - Sum of lives of all these nuclei = tλNdt.
    - Therefore average life of the sample  $=t_{av}=(_{0}^{\infty}\int(t\lambda N)dt/N_{0})$
    - $\circ = (\lambda/N_0) \, {}_0^{\infty} \int (tN) \, dt.$
    - o From exponential law of decay :-  $N = N_0 e^{-\lambda t}$
    - $\circ t_{av} = (\lambda/N_0) \int tN_0 e^{-\lambda t} dt = (\lambda/N_0) \int t e^{-\lambda t} dt$
    - $\circ t_{av} = \lambda_0^{\infty} \int t e^{-\lambda t} dt$
    - $\circ = \lambda [t e^{-\lambda t} dt ((e^{-\lambda t}.1) dt)]$
    - $\circ = \lambda [(te^{-\lambda t})/(-\lambda)]_0^{\infty} \lambda [(e^{-\lambda t})/(-\lambda)]_0^{\infty}$
    - $\circ = \lambda[0] \lambda[(e^{-\lambda t})/\lambda^2]_0^{\sim}$
- o Using  $t_{(1/2)} = (0.693/\lambda)$  and  $t_{av} = (1/\lambda)$
- $\circ$   $t_{av} = (t_{(1/2)}/0.693)$

#### Problem:-

A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

### Answer:-

Half-life of the radioactive isotope = T years

Original amount of the radioactive isotope =  $N_0$ 

(a) After decay, the amount of the radioactive isotope = N

It is given that only 3.125% of N<sub>0</sub> remains after decay. Hence, we can write:

 $(N/N_0)$ = 3.125 %=( 3.125)/ (100) = (1/32)

```
But (N/N_0) = e^{-\lambda t}
Where, \lambda = Decay, constant t = Time
Therefore e^{-(\lambda t)} = (1/32)
-(\lambda t) = \ln(1) - \ln(32)
-(\lambda t) = 0 - 3.4657
t = (3.4657)/(\lambda)
Since \lambda = (0.693)/(T)
Therefore t = (3.466)/((0.693)/(T)) \approx 5T years.
Hence, the isotope will take about 5T years to reduce to 3.125% of its original value.
(b) After decay, the amount of the radioactive isotope = N
It is given that only 1% of N<sub>0</sub> remains after decay. Hence, we can write:
(N/N_0)=1 \%=(1/100)
But (N/N_0) = e^{-\lambda t}
Therefore e^{-(\lambda t)} = (1/100)
-(\lambda t) = \ln(1) - \ln(100)
-(\lambda t) = 0 - 4.6052
t = (4.6052)/(\lambda)
Since \lambda = (0.693)/(T)
Therefore t = (4.6052)/((0.693)/(T)) = 6.645T years.
Hence, the isotope will take about 6.645T years to reduce to 1% of its original value.
<u>Problem:-</u> Obtain the amount (<sup>60</sup><sub>27</sub>Co) of necessary to provide a radioactive source of 8.0 mCistrength. The half-life
     of \binom{60}{27}Co) is 5.3 years.
Answer:- The strength of the radioactive source is given as:
(dN/dt) = 8.0 mCi
=8x10^{-3}x 3.7x10^{10}
=29.6 \times 10^{7} \text{ decay/s}.
Where,
N = Required number of atoms
Half-life of (^{60}_{27}Co), T_{1/2} = 5.3 years
=5.3\times365\times24\times60\times60
= 1.67 \times 10^8 \,\mathrm{s}
For decay constant \lambda, we have the rate of decay as:
(dN/dt) = (\lambda N)
Where \lambda = (0.693/T_{1/2}) = (0.693)/(1.67 \times 10^8) \text{ s}^{-1}
Therefore N = (1/\lambda) (dN/dt)
= (29.6 \times 10^7)/(0.693)/(1.67 \times 10^8) = 7.133 \times 10^{16} atoms
For (<sup>60</sup><sub>27</sub>Co):
Mass of 6.023 \times 10^{23} (Avogadro's number) atoms = 60 g
Therefore, Mass of (7.133 \times 10^{16}) atoms= (60x7.1333x10^{16})/(6.023x10^{23})
=7.106 \times 10^{-6} g
Hence, the amount of \binom{60}{27}Co) necessary for the purpose is 7.106 \times 10^{-6} g.
```

### Alpha decay

- o In alpha decay α particles are emitted. Daughter nucleus is formed from the parent nucleus.
- The atomic number decreases by 2 and Mass number increases by 4.
- o It is a very spontaneous process and happens on its own.
- It occurs only in radionuclides.
- o For example:-
  - $\circ$   $^{238}_{92}U$  (unstable) -->  $_{90}^{234}Th + _{2}^{4}He$ 
    - o (Uranium (U) is known as parent nucleus and Thorium (Th) is known as daughter nucleus).
    - $\circ$  <sup>4</sup>He is  $\alpha$  particle.
  - o General form of alpha decay:  $-{}^{A}_{z}X$  (parent)  $a^{(A-4)}_{(Z-2)}Y + {}^{4}_{2}He$ 
    - o Where Y is daughter nucleus.

### Q-value of alpha decay

- Q-value is a parameter or a characteristic of a nuclear reaction which describes whether the reaction can take place or not.
- o Q-value is defined as difference in the initial mass energy and final mass energy of decayed products.
- o Consider the general equation:-
  - $\circ$   $^{A}_{z}X$  (parent) --> $^{(A-4)}_{(Z-2)}Y + _{2}^{4}$
  - o Initial rest mass energy  $U_i = [m(^{A}_{Z}X) Zm_{e}]c^{2}$ 
    - $\circ$  Where m ( $^{A}_{Z}X$ ) = rest mass and Zme = mass of electrons.
  - $_{\text{O}}$  Final rest mass energy  $U_f = [m(^{(A-4)}_{(Z-2)} Y (Z-2) m_e + m(_2^4 He) 2m_e] c^2$
  - $\circ$  Q =U<sub>i</sub> U<sub>f</sub>
  - $\circ = [m \binom{A_2X}{-2} 2m_e m \binom{(A-4)}{(Z-2)} + (Z-2) m_e m \binom{A_2}{-4} + 2m_e]c^2$
  - $O = [m (^{A}_{z}X) m (^{(A-4)}_{(z-2)}Y m (_{2}^{4}He)]c^{2}]$
- If Q =(+ive) then equation is energetically allowed.
  - $\circ$  => Q>0, (U<sub>i</sub>-U<sub>f</sub>) >0=> U<sub>i</sub>>U<sub>f</sub>
  - This implies energy of the reactants is more than the energy of the products.
  - o There is some free energy that is in the form of kinetic energy.
- If Q =(-ive) then equation is not energetically allowed.

Problem: - Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of (a)  $\binom{226}{88}$ Ra) and (b)  $\binom{220}{86}$ Rn). Given m  $\binom{226}{88}$ Ra) =226.0250 u, m  $\binom{220}{86}$ Rn) = 220.01337u, m  $\binom{222}{86}$ Rn) =222.01750u, m  $\binom{216}{84}$ Po) =216.00189u.

#### Answer:

(a) Alpha particle decay of  $(^{226}_{88}Ra)$  emits a helium nucleus. As a result, its mass number reduces to (226 - 4) 222 and its atomic number reduces to (88 - 2) 86. This is shown in the following nuclear reaction.

$$\binom{226}{88}$$
Ra) -->  $\binom{222}{88}$ Ra) +  $\binom{4}{2}$ He)

Q-value of emitted  $\alpha$ -particle = (Sum of initial mass – Sum of final mass)

 $c^2$  Where, c =Speed of light It is given that:

$$m(^{226}_{88}Ra) = 226.0250 u, m(^{222}_{86}Rn) = 222.01750u, m(^{4}_{2}He) = 4.002603u$$

Q-value =  $[226.02540 - (222.01750 + 4.002603)] \text{ u c}^2$ 

 $= 0.005297 \text{ u c}^2$ 

But 1 u =  $931.5 \, (MeV/c^2)$ 

Therefore Q =  $(0.005297 \times 931.5) \approx 4.94 \text{ MeV}$ 

Kinetic energy of the  $\alpha$ -particle =

(Mass number after decay) / (Mass number before decay) x Q

```
= (222)/ (226) x 4.94 = 4.85MeV.

(b) Alpha particle decay of (^{220}{}_{86}Rn) (^{220}{}_{86}Rn) --> (^{216}{}_{84}Po) + (^{4}{}_{2}He) It is given that:

Mass of (^{220}{}_{86}Rn) = 220.01137 u

Mass of (^{216}{}_{84}Po) = 216.00189 u

Therefore, Q-value = [220.01137 – (216.00189+4.002603)] x931.5 \approx 641 MeV

Kinetic energy of the \alpha-particle = (220-4)/ (220) x 6.41 = 6.29 MeV
```

<u>Problem:</u> -We are given the following atomic masses:  $^{238}_{92}$ U = 238.05079 u  $^{4}_{2}$ He = 4.00260 u,  $_{90}^{234}$ Th = 234.04363 u,  $^{1}_{1}$ H= 1.00783 u,  $^{237}_{91}$ Pa = 237.05121 u Here the symbol Pa is for the element protactinium (Z = 91).

- (a) Calculate the energy released during the alpha decay of <sup>238</sup><sub>92</sub>U.
- (b) Show that <sup>238</sup><sub>92</sub>U cannot spontaneously emit a proton.

#### Answer:-

(a) The alpha decay of  $^{238}_{92}$ U is given by equation  $^{A}_{Z}$  X -->  $^{(A-4)}_{(Z-2}$ Y +  $^{4}_{2}$ He

The energy released n this process is given by:

$$Q = (M_U - M_{Th} - M_{He}) c^2$$

Substituting the atomic masses as given in the data, we find

$$Q = (238.05079 - 234.04363 - 4.00260) u \times c^2$$

- $= (0.00456 \text{ u}) \text{ c}^2$
- = (0.00456 u) (931.5 MeV/u)
- = 4.25 MeV.

(b) If  $^{238}_{92}$  U spontaneously emits a proton, the decay process would be  $^{238}_{92}$  U -->  $^{237}_{91}$  Pa +  $^{1}_{\ 1}$  H

The Q for this process to happen is

- $= (M_U M_{Pa} M_H) c^2$
- =  $(238.05079 237.05121 1.00783) \text{ u} \times \text{c}^2$
- $= (-0.00825 \text{ u}) \text{ c}^2$
- = -(0.00825 u) (931.5 MeV/u)
- = -7.68 MeV

Thus, the Q of the process is negative and therefore it cannot proceedspontaneously. We will have to supply energy of 7.68 MeV to a<sup>238</sup><sub>92</sub>U nucleus to make it emit a proton.

#### **Beta decay**

- o In case of beta decay either electron or positron is emitted.
- Mass number remains the same.
- β-decay --> Electron is emitted.
  - Atomic number increases by 1.
  - For example:- $^{32}_{15}$ P  $\grave{a}_{16}^{32}$ S +  $\acute{e}^{-}$  + $\stackrel{\cdot}{\nabla}$  where  $\stackrel{\cdot}{\nabla}$  = anti-neutrino
  - Q-value for β̄decay:-
    - $\circ {}^{A}_{Z}X\dot{a}^{A}_{(Z+1)}Y + e^{-}+V$
    - o Initial rest mass energy  $U_i = [m ({}^{A}_{Z}X) Zm_e]c^2$

- $\circ$  Where m  $\binom{A}{Z}X$ ) = rest mass and Zme = mass of electrons
- Final rest mass energy  $U_f = [m(^A_{(Z+1)} Y) (Z+1) m_e + m_e] c^2$
- $\circ = [m(^{A}_{(Z+1)} Y) Zm_{e}] c^{2}$
- o Therefore Q =U<sub>i</sub>− U<sub>f</sub>
- $\circ = [m(^{A}_{Z}X) Zm_{e} m(^{A}_{(Z+1)}Y) + Zm_{e}] c^{2}$
- $\circ$  Q = [m( $^{A}_{z}X$ ) m( $^{A}_{(z+1)}Y$ )]

### Example:-

- $\circ$   $\beta^{\dagger}$  decay à Positron(same as electron but with (+ ive) charge) is emitted.
  - Atomic number decreases by 1.
  - o For example:-  $^{23}_{11}$ Na --> $_{10}^{22}$ Ne + e<sup>+</sup>+v where v = neutrino
  - Q-value for beta (+ive) decay:-
    - $\circ {}^{A}_{Z}X --> {}^{A}_{(Z-1)}Y + e^{+} + v$
    - Initial rest mass energy U<sub>i</sub> = [m (<sup>A</sup><sub>Z</sub>X) Zm<sub>e</sub>]c<sup>2</sup>
      - Where  $m\binom{A}{Z}X$  = rest mass and Zme = mass of electrons
    - Final rest mass energy  $U_f = [m(^A_{(Z-1)} Y) (Z-1) m_e + m_e] c^2$
    - o Therefore Q = U<sub>i</sub> U<sub>f</sub>
    - $\circ = [m \binom{A_{Z}X}{-Zm_{e}} m\binom{A_{(Z-1)}Y}{-(Z-1)m_{e}} m_{e}] c^{2}$
    - $O = [m(^{A}_{Z}X) m(^{A}_{(Z-1)}Y) 2 m_{e}] c^{2}$
  - o Example to show whether nuclear reaction can take place or not:-
    - $\circ$   $^{1}_{1}p(proton)$   $\dot{a}_{0}^{1}n(neutron) + e^{+} + v$ . The mass of neutron is greater than the mass of proton.
    - The Q-value = (-ive) as a result the above reaction is not possible. This means conversion of stable proton to neutron is not allowed.
    - o Consider  $_0^1$ n (neutron)  $a_1^1$ p (proton) +  $e^-$  + v
    - The Q-value =(+ive) as a result above reaction is possible. This means conversion of neutron to proton is allowed.

#### Beta-decay: neutrino and anti-neutrino

- o Neutrino (v) and anti-neutrino (v) are neutral particles.
- They also have negligible mass.
- o They have extremely high penetration power.
- o They have extremely weak interaction with matter.

<u>Problem:-</u> The nucleus ( $^{23}_{10}$ Ne) decays by  $\beta^{-}$  emission. Write down the  $\beta$  decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:m ( $^{23}_{10}$ Ne) = 22.994466 u,m ( $^{23}_{11}$ Na) = 22.989770 u.

Answer:- In  $\beta$  emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

 $\beta^{-}$  Emission of the nucleus ( $^{23}_{10}$ Ne)

$$\binom{23}{10}$$
Ne) -->  $\binom{23}{11}$ Na) +  $e^{-}$  +  $-$ v + Q

It is given that:

Atomic mass m ( $^{23}_{10}$ Ne) of = 22.994466 u

Atomic mass m  $\binom{23}{11}$ Na) of = 22.989770 u

Mass of an electron,  $m_e = 0.000548 u$ 

Q-value of the given reaction is given as:

 $Q = [m (^{23}_{10}Ne) + [m (^{23}_{11}Na) + m_e]] c^2$ 

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There are 10 electrons in and 11 electrons in (<sup>23</sup><sub>11</sub>Na). Hence, the mass of the electron is cancelled in the Q-value equation.

Therefore Q = [22.994466 - 22.989770] c<sup>2</sup>

 $= (0.004696 c^2) u.$ 

But  $1u = 9.31 \, (MeV/c^2)$ 

Therefore  $Q = (0.004696 \times 931.5) = 4.374 MeV$ .

The daughter nucleus is too heavy as compared to e<sup>-</sup> and <sup>-</sup>v. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the Q-value, i.e., 4.374 MeV.

<u>Problem:-</u> For the  $\beta^+$  (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K-shell, is captured by the nucleus and a neutrino is emitted).  $e^+ + {}^A_Z X - -> {}^A_{(Z-1)} Y + v$ 

Show that  $\beta^{\dagger}$  if emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

#### Answer:-

Let the amount of energy released during the electron capture process be Q<sub>1</sub>. The nuclear reaction can be written as:

$$e^+ + {}^A_Z X --> {}^A_{(Z-1)} Y + v + Q_1$$
 (equation(1))

Let the amount of energy released during the positron capture process be Q<sub>2</sub>. The nuclear reaction can be written as:

 $_{z}^{A}X --> _{(Z-1)}^{A}Y + e^{+} + v + Q_{2}$  (equation (2))

 $m_N (^A_Z X) = Nuclear mass of (^A_Z X)$ 

 $m_N(^A_{(Z-1)}Y) = Nuclear mass of (^A_{(Z-1)}Y)$ 

 $m(_{z}^{A}X) = Atomic mass of (_{z}^{A}X)$ 

 $m(^{A}_{(Z-1)} Y) = Atomic mass of (^{A}_{(Z-1)} Y)$ 

m<sub>e</sub> = Mass of an electron

c = Speed of light

Q-value of the electron capture reaction is given as:

 $Q_1 = [m_N(^{A}_{Z}X) + m_e - m_N(^{A}_{(Z-1)}Y)] c^2$ 

=  $[m (_{Z}^{A}X) - Zm_{e} + m_{e} - m (_{(Z-1)}^{A}Y) + (Z-1) m_{e}] c^{2}$ 

=  $[m(^{A}_{Z}X) - m^{A}_{(Z-1)}Y]c^{2}$  (equation (3))

Q-value of the positron capture reaction is given as:

 $Q_2=m_N(^{A}_{Z}X) - m_N(^{A}_{(Z-1)}Y) - m_e] c^2$ 

=  $[m (_{Z}^{A}X) - Zm_{e} - m (_{(Z-1)}^{A}Y) + (Z-1) m_{e} - m_{e}] c^{2}$ 

=  $[m \binom{A}{Z}X] - m^A_{(Z-1)}Y - 2m_e] c^2$  (equation (4))

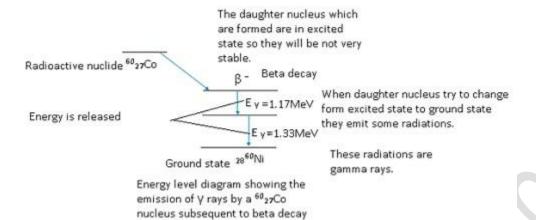
It can be inferred that if  $Q_2 > 0$ , then  $Q_1 > 0$ ; Also, if  $Q_1 > 0$ , it does not necessarily mean that  $Q_2 > 0$ .

In other words, this means that if  $\beta^{\dagger}$  emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is because the Q-value must be positive for an energetically-allowed nuclear reaction.

#### Gamma decay

- o In Gamma decay γ rays are emitted. γ rays are electromagnetic waves with short wavelength.
- o Most of the daughter nuclei of alpha decay and beta decay are in excited state.
- As a result they are unstable. When the daughter nuclides try to transit from excited state to ground state they
  emit radiations.

These radiations are known as γThis known as gamma decay.

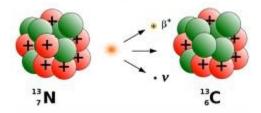


Comparison of Alpha, Beta & Gamma Rays

Companson of Alpha, beta & Gamma Rays			
Alpha Rays		Beta Rays	Gamma Rays
1.	Stream of alpha particles they are known as alpha rays.	Stream of electrons     they are known as     beta rays.	<ol> <li>Gamma rays are Electromagnetic radiation.</li> </ol>
2.	Alpha rays get deflected by electric field as they are positively charged.	2.Beta rays get deflected by electric field as they are negatively charged.	2. Gamma rays are undeflected by the electric field as they are neutral.
3.	Alpha rays get deflected by magnetic field as they are positively charged	Beta rays get deflected     by magnetic field as     they are negatively     charged	3. Gamma rays are undeflected by the magnetic field as they are neutral
4.	Alpha rays have low penetrating power.	4.Beta rays have high penetrating power.	4.Gamma rays have largest penetrating power.
5.	lonizing power of alpha rays is high.	5.lonizing power of beta rays is lesser than alpha rays.	5.lonizing power of gamma rays is least.

### **Nuclear Energy**

- Nuclear energy is the energy that holds together the nuclei of atoms.
- o Nuclear energy is obtained from nucleusby either:
  - o Breaking of heavy nucleus into 2 relatively lighter nuclei known as <u>nuclear fission</u>or by
  - o Combining 2 lighter nuclei to form a heavy nucleus known as <u>nuclear fusion</u>.
- o Nuclear energy is becoming apossible solution for the energy crisis in the world.
- Electric energy can be harnessed from nuclear energy.



### **Nuclear reactor**



### **Types of Nuclear reactions**

Nuclear fission

Heavy Nuclei

Breaking into

Lighter nuclei

Large amount of energy is

released

<u>Problem:-</u> A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that thecoin is entirely made of ( $_{29}^{63}$ Cu) atoms (of mass 62.92960 u).

Answer: - Mass of a copper coin, m' = 3 g

Atomic mass of  $\binom{63}{29}$  Cu) atom, m = 62.92960 u

The total number of  $\binom{63}{29}$ Cu) atoms in the coin, N =  $\binom{N_A \times m'}{Mass}$ 

Where,

 $N_A = Avogadro's number = 6.023 \times 10^{23} atoms /g$ 

Mass number = 63 g

Therefore N =  $(6.023 \times 10^{23} \text{ x3})/(63) = 2.868 \times 10^{22} \text{ atoms.}$ 

 $\binom{63}{29}$  Cu) nucleus has 29 protons and (63-29) = 34 neutrons.

Therefore, Mass defect of this nucleus,  $\Delta m' = 29 \times m_H + 34 \times m_n - m$ 

Where,

Mass of a proton,  $m_H = 1.007825 u$ 

Mass of a neutron,  $m_n = 1.008665 u$ 

Therefore,  $\Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$ 

= 0.591935 u

Mass defect of all the atoms present in the coin,  $\Delta m = 0.591935 \times 2.868 \times 10^{22}$ 

=1.69766958x10<sup>22</sup>u.

But 1 u =  $931.5 \, (MeV/c^2)$ 

Therefore,  $\Delta m = 1.69766958 \times 10^{22} \times 931.5 \text{ (MeV/c}^2\text{)}$ 

Hence, the binding energy of the nuclei of the coin is given as:

 $E_b = \Delta mc^2$ 

 $= 1.69766958 \times 10^{22} \times 931.5 (MeV/c^2)/c^2$ 

 $= 1.581 \times 10^{25} \text{ MeV}$ 

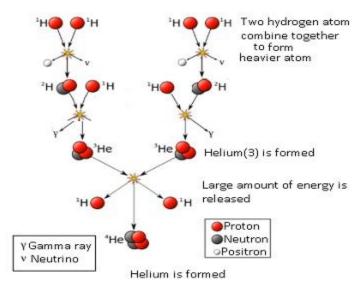
But 1 MeV =  $1.6 \times 10^{-13}$  J

 $E_h = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$ 

 $= 2.5296 \times 10^{12} \,\mathrm{J}$ 

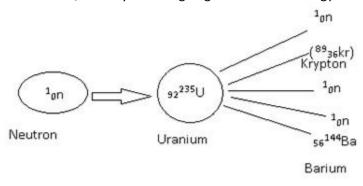
This much energy is required to separate all the neutrons and protons from the given coin.

Nuclear fusion



#### **Nuclear Fission**

- Scientist named Fermi was the one who performed first nuclear fission reaction.
- He considered the sample of  $_{92}^{235}$ U and bombarded it with a neutron  $_0^1$ n i.e.  $_0^1$ n +  $_{92}^{235}$ U(Target nuclei)à $_{92}^{236}$ U(unstable so broke down into) --> $_{56}^{144}$ Ba +  $_{36}^{89}$ kr +3 $_0^1$ n
  - $\circ$  Where  $\binom{1}{56}^{144}$ Ba) = Barium,  $\binom{89}{36}$ kr)=Krypton and  $\binom{1}{0}$ n) = neutron.
  - This reaction is known as nuclear fission reaction as heavier nuclei is broken down into lighter nuclei ,there by releasing large amounts of energy.

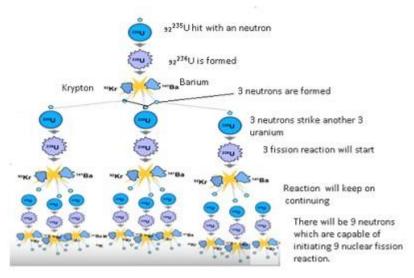


Huge amount of energy released

#### Types of Nuclear Fission reactions:

- 1. Uncontrolled chain reaction:-
  - 1. In uncontrolled chain reaction, the reaction is uncontrolled and rapid. The reaction keeps increasing and becomes huge.
  - 2. Energy of the order of mega electron volts is produced.
  - 3. This reaction is known as chain reaction as the product formed in first reaction initiates the second reaction and so on.
  - 4. One of the important applications of this reaction is in making atom bombs, hydrogen bombs and nuclear bombs.
  - 5. (Number of neutrons hitting next target)/ (number of neutrons emitted)>=1.
  - 6. Reaction that takes place :-
    - $^{1}_{0}$ n +  $^{235}_{92}$ U --> $^{236}_{92}$ U(unstable so broke down into) --> $^{144}_{56}$ Ba +  $^{89}_{36}$ kr +3  $^{1}_{0}$ n

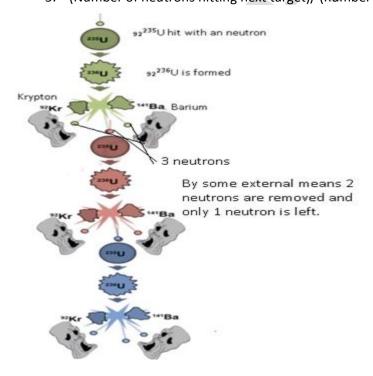
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Huge amount of energy will be released in this reaction

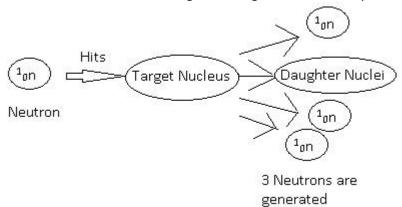
#### 2. Controlled chain reaction:-

- 1. In controlled chain reaction, the reaction is controlled and steady.
- 2. Reaction that takes place :-
  - 1.  $^{1}_{0}$ n +  $^{235}_{92}$ U -->  $^{236}_{92}$ U(unstable so broke down into) --> $^{144}_{56}$ Ba +  $^{89}_{36}$ kr +3  $^{1}_{0}$ n
  - 2. Using some methods 2 neutrons are removed and only 1 neutron is allowed to hit the next target.
- 3. Energy released is less as compared to the energy released in the nuclear fission reaction.
- 4. One of the most important applications is in Nuclear Reactor where electricity can be produced.
- 5. (Number of neutrons hitting next target)/ (number of neutrons emitted)<1.



#### **Nuclear Reactor**

- Nuclear reactors are useful in producing electricity.
- o A nuclear reactor is an arrangement to generate electricity whichmakes use of nuclear fission.

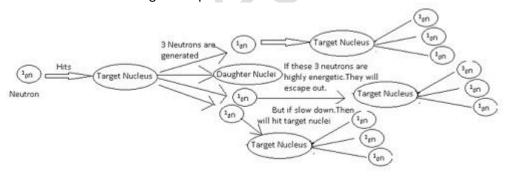


### Requirements for controlled nuclear fission in reactor:

- 1. Neutrons to be slowed down.
- Neutrons are slowed down by using <u>moderators</u> which are lighter nuclei which slow down fast moving neutrons by elastic collision.
- Commonly used moderators:-

#### 1.Water

- 2. Heavy Water
- 3. Graphite
- o Consequence of use of moderators:-
  - Multiplication factor of neutrons increases: When a neutron hits a target nucleus along with daughter nucleus, it produces 3 neutrons. These 3 neutrons are highly energetic but they need to be slowed down, so they can hit the target nucleus.
  - As a result high multiplication factor results in uncontrolled chain reaction.

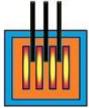


- Excess neutrons to be absorbed.
- As uncontrolled chain reaction is wanted therefore to absorb excess neutrons Control Rods are used.
- o These control rods are inserted in the core of the nuclear reactor.
- Control rods are capable of initiating(while taking out of the reactor) and stopping(inserting in the nuclear reactor) the nuclear reaction.
- o As they absorb all the excess neutronsthere are no neutrons left to start the reaction.
- Control rods are made up of neutron absorbing materials.
- They decrease the multiplication factor of neutrons to a very small value.

Commonly used material is <u>Cadmium</u>.

#### **Control Rods**





### Construction of Nuclear Reactor

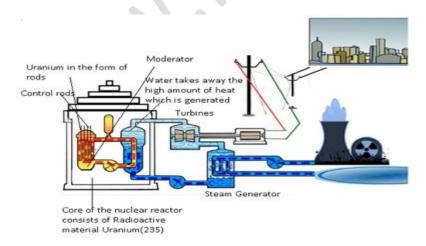
- The core of the nuclear reactor consists of uranium (<sup>235</sup>U) in the form of cylindrical rods. These rods are dipped inside a liquid which is the moderator.
- Whenever one neutron strikes this uranium rod nuclear fission reaction starts and 3 fast moving neutrons are produced.
- Because of the moderator these 3 neutrons undergo elastic collision as a result they slow down before they strike the second rod.
- Geometry of the core is such that only one out of 3 neutrons which are emitted strike the next rod making the reaction a controlled one.
- When the control rods are inserted inside they will absorb all the extra neutrons. Since there are no neutrons nuclear fission reaction will stop.
- Large amount of energy is also released in the core.
- In order to extract the energy from the core water at very high pressure is passed through it.
- As hot water passes through it produces steam in the steam generators.
- o This steam is used to run the turbines which in turn produce electricity.
- This process will keep on continuing till the uranium on the rods does not get over. Then the rods have to be replaced in the nuclear reactor.

### Advantages:-

- 1. Energy released is extremely large.
- 2. Needs fuel in extremely small quantity.

#### **Disadvantages**:-

- 1. Spent fuel is highly radioactive and extremely hazardous to all life forms.
- 2. Accumulation of radioactive waste.



<u>Problem:-</u> A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much (<sup>235</sup><sub>92</sub>U) did it containinitially? Assume that the reactor operates 80% of the time that all the energy generatedarises from the fission of (<sup>235</sup><sub>92</sub>U) and that this nuclide is consumed only by the fissionprocess.

<u>Answer:-</u> Half-life of the fuel of the fission reactor,  $t_{(1/2)}$ = 5 years

 $= 5 \times 365 \times 24 \times 60 \times 60 \text{ s}$ 

We know that in the fission of 1 g of (235 g2U) nucleus, the energy released is equal to 200MeV.

1 mole, i.e., 235 g of ( $^{235}_{92}$ U) contains (6.023 × 10 $^{23}$ ) atoms.

 $1 g \binom{235}{92} U$  contains =  $(6.023 \times 10^{23}) / (235)$  atoms.

The total energy generated per gram of (235 92 U) is calculated as:

 $E = ((6.023 \times 10^{23})/(235)) \times 200 \text{MeV/g}$ 

=  $(200x6.023x10^{23}x1.6x10^{-19}x10^6)/(235) = 8.20x10^{10} J/g$ 

The reactor operates only 80% of the time.

Hence, the amount of (235 92 U) consumed in 5 years by the 1000 MW fission reactor iscalculated as:

=  $(5x80x60x60x365x24x1000x10^6)$  g /  $(100x8.20x10^{10})$ 

=1538kg

Therefore, Initial amount of  $\binom{235}{92}U$  = 2 × 1538 = 3076 kg

<u>Problem:-</u> The fission properties of (<sup>239</sup><sub>94</sub>Pu) are very similar to those of (<sup>235</sup><sub>92</sub>U). The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure (<sup>239</sup><sub>94</sub>Pu) undergo fission?

<u>Answer:-</u> Average energy released per fission of (<sup>239</sup><sub>94</sub>Pu), E<sub>av</sub>=180MeV

Amount of pure ( $^{239}_{94}$ Pu), m = 1 kg = 1000 g

 $N_A$ = Avogadro number =  $6.023 \times 10^{23}$ 

Mass number of  $\binom{239}{94}$ Pu) = 239 g

1 mole of ( $^{239}_{94}$ Pu) contains N<sub>A</sub> atoms.

Therefore mg of  $\binom{239}{94}$ Pu) contains  $((N_A)/(Mass number) \times m)$  atoms

=  $((6.023 \times 10^{23})/(239) \times 1000) = 2.52 \times 10^{24}$  atoms.

Therefore Total energy released during the fission of 1 kg of ( $^{239}_{\phantom{194}P4}$ Pu) is calculated as:

 $E=E_{av} \times 2.52 \times 10^{24}$ 

 $=180x2.52x10^{24} = 4.536x10^{26}MeV$ 

Hence,  $4.536x10^{26}$  MeV is released if all the atoms in 1 kg of pure ( $^{239}_{94}$ Pu) undergo fission.

#### **Nuclear Fusion**

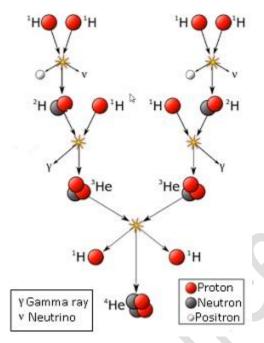
- o In a nuclear fusion reaction two lighter nuclei combine to form a relatively heavier nucleus.
- o In this process huge amount of energy is also released.
- o Temperature at which protons would have enough energy to overcome the coulomb's barrier is very high.

### **Thermonuclear fusion**

- o Increasing the temperature of the material until the particles have enough energy due to their thermal motions alone –to overcome the coulomb barrier.
- For thermonuclear fusion, extreme conditions of temperature and pressure are required.
- Example of Thermonuclear fusion is generation of energy in stars.
  - For example: From the sun we get large amount of energy and this energy generated due to the thermonuclear fusion reaction taking place in the sun.

### **Energy generation in sun**

- o In sun the energy generation is a multi- step process. There are total of 4 steps involved in the energy generation inside sun.
  - $\circ$  Step1:-  ${}^{1}_{1}$ H(proton) +  ${}^{1}_{1}$ H(proton) -->  ${}^{2}_{1}$ H(deuteron)+ e<sup>+</sup>(positron) + v(neutrino)+0.42MeV
  - Step2:- e<sup>+</sup>(positron) + e<sup>-</sup>(electron) -->γ(Gamma rays) + 1.02MeV
  - $\circ$  Step3:-  $^{2}_{1}$ H(deuteron)+  $^{1}_{1}$ H(proton) --> $^{3}_{2}$ He(helium) + y(Gamma rays)+5.49MeV
  - $\circ$  Step4:-  ${}^{3}_{2}H + {}^{3}_{2}H --> {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H + 12.86MeV$
  - o Step 1,2, and 3 occur twice in the sun and the step 4 occurs only once.
- When all the above 4 reactions are combined together then four hydrogen atoms combine to form a<sup>4</sup><sub>2</sub>He atom with a release of 26.7MeV of energy.
- Final reaction is:-  $4^{1}_{1}H + 2e^{--}_{2}He + 6 y + 2 v + 26.7MeV$
- o It is also known as proton-proton cycle because this process starts with protons.



<u>Problem:-</u> How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as  ${}^{2}_{1}H + {}^{2}_{1}H --> {}^{3}_{2}He+n+3.27MeV$ 

Answer:- The given fusion reaction is:

 $^{2}_{1}H + ^{2}_{1}H --> ^{3}_{2}He+n+3.27MeV$ 

Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains  $6.023 \times 10^{23}$  atoms.

Therefore, 2.0 kg of deuterium contains=  $((6.023 \times 10^{23})/(2)) \times (2000) = 6.023 \times 10^{26} \text{ atoms}$ 

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27

MeV energy is released.

Therefore, total energy per nucleus released in the fusion reaction:

 $E = (3.27/2) \times 6.023 \times 10^{26} \text{ MeV}$ 

=  $(3.27/2) \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$ 

 $=1.576 \times 10^{14} \text{ J}$ 

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

- $= (1.576x10^{14})/(100x60x60x24x365)$
- $= (4.9 \times 10^4)$  years.

<u>Problem:-</u> From the relation  $R = R_0 A^{1/3}$ , where  $R_0$  is a constant and A is the mass number of anucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

**Answer:-** We have the expression for nuclear radius as:

 $R = R_0 A^{1/3}$ 

Where,

 $R_0$  = Constant.

A = Mass number of the nucleus

Nuclear matter density,  $\rho = (Mass of the nucleus)/(volume of nucleus)$ 

Let m be the average mass of the nucleus.

Hence, mass of the nucleus = mA

Therefore  $\rho = (mA)/((4/3)\pi R^3) = (3mA)/(4\pi)(R_0A^{(1/3)})^3 = (3m)/(4\pi R_0^3)$ 

Hence, the nuclear matter density is independent of A. It is nearly constant.